Hope, Fear, and Aspiration

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Based on the joint work with Prof. Xunyu Zhou in Oxford

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  - Lopes' SP/A theory (Lopes 1987, Lopes and Oden 1999)

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- Inspired by these models of choice, we propose a new portfolio choice model featuring *hope, fear* and *aspiration*
- This model is based on SP/A theory and rank-dependent utility
- Analytical solutions are found and the impact of hope, fear, and aspiration is studied

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- Hope and fear coexist. Hope is optimism over extremely satisfactory situations and fear is pessimism over really poor situations
- When evaluating a random prospect, hope makes the agent overweight the probability of best outcomes, while fear makes him overweight the probability of worst outcomes (rank-dependent utility theory and prospect theory)

Hope and Fear (Con'd)

- We appeal to rank-dependent utility to model hope and fear

$$
\begin{aligned}
V(X) :&= \int_0^{+\infty} u(x) d[-w(1 - F_X(x))] \\
&= \int_0^{+\infty} u(x) w'(1 - F_X(x)) dF_X(x)
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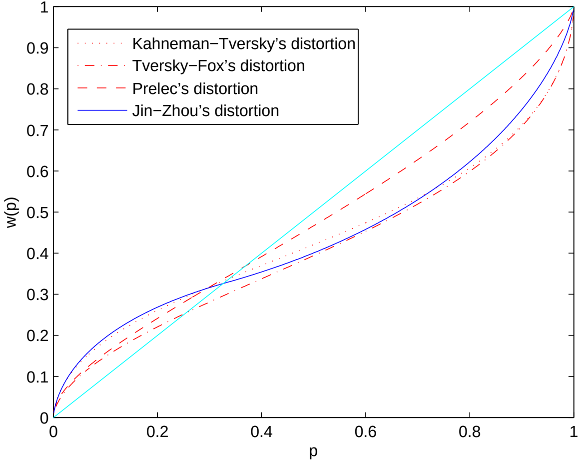
- $w(\cdot)$, *probability distortion function*, is usually reverse-S shaped, capturing hope and fear simultaneously

Distortion Functions



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- It reflects the opportunities at hand as well as the constraints imposed by the environment
  - The direct assessment of what is reasonable or safe to hope for
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- Aspiration is modeled by the constraint

$$P(X \geq A) \geq \alpha$$

where A is *aspiration level*, α is *confidence level*

Literature

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- Carlier and Dana (2009) considered the portfolio selection problem with rank-dependent utility

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- A static problem

Portfolio Choice Model

- *HF/A portfolio choice model*

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- Non-concavity due to the distortion function
- Quantile formulation (Schied 2004, Carlier and Dana 2006, Jin and Zhou 2008, He and Zhou 2010)

Quantile Formulation

- The HF/A portfolio choice model

$$\begin{array}{ll} \underset{X}{\text{Max}} & V(X) := \int_0^{+\infty} u(x) d[-w(1 - F_X(x))] \\ \text{Subject to} & X \in \mathcal{F}(T), X \geq 0, P(X \geq A) \geq \alpha, \\ & E[\rho X] \leq x_0 \end{array} \qquad \text{(P)}$$

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- Objective

$$V(X) = \int_0^1 u(F_X^{-1}(z)) w'(1-z) dz$$

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- Budget constraint (by Hardy-Littlewood inequality)

$$\int_0^1 F_\rho^{-1}(1-z)F_X^{-1}(z)dz \leq x_0$$

Quantile Formulation (Cont'd)

- Let $G(\cdot) = F_X^{-1}(\cdot)$, we derive the quantile formulation

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- If $G^*(\cdot)$ is an optimal solution to the quantile formulation, then $X^* := G^*(1 - F_\rho(\rho))$ is an optimal terminal wealth

Lagrange Dual Method

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- Explicit solutions can be found

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- Carlier and Dana (2009) derive a similar result

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  then there exists $c > 0$ such that $X_1^* > X_2^*$ on $\{\rho \le c\}$.
- Higher hope index makes the agent gamble more on the best scenarios so that he can get higher payoff when these scenarios really happen.

Lottery-likeness Index

- *Lottery-likeness index*

$$\mathcal{L}(A) = \frac{\text{essinf}\left(X^* \mid \rho \le F_\rho^{-1}(\alpha)\right)}{\text{esssup}\left(X^* \mid \rho > F_\rho^{-1}(\alpha)\right)}.$$

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- **Theorem:** $\mathcal{L}(A)$ can be computed explicitly and is increasing w.r.t the aspiration level A

Example

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- Jin-Zhou distortion

$$w(z) = \begin{cases} k e^{b\mu_\rho + (a+b)\sigma_\rho \Phi^{-1}(1-z_0) + \frac{(a\sigma_\rho)^2}{2}} \Phi\left(\Phi^{-1}(z) + a\sigma_\rho\right), & z \le 1 - z_0, \\ A + k e^{b\mu_\rho + \frac{(b\sigma_\rho)^2}{2}} \Phi\left(\Phi^{-1}(z) - b\sigma_\rho\right), & z \ge 1 - z_0 \end{cases}$$

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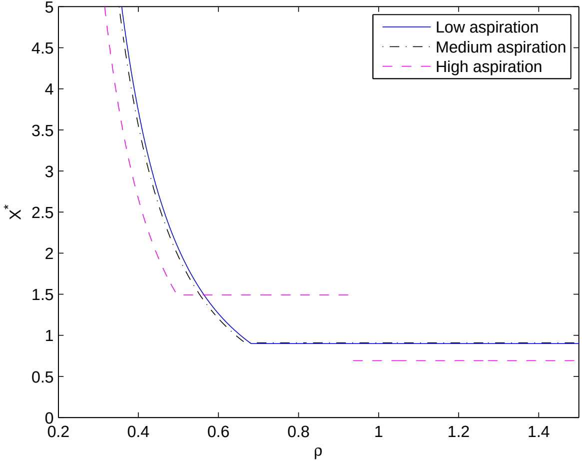
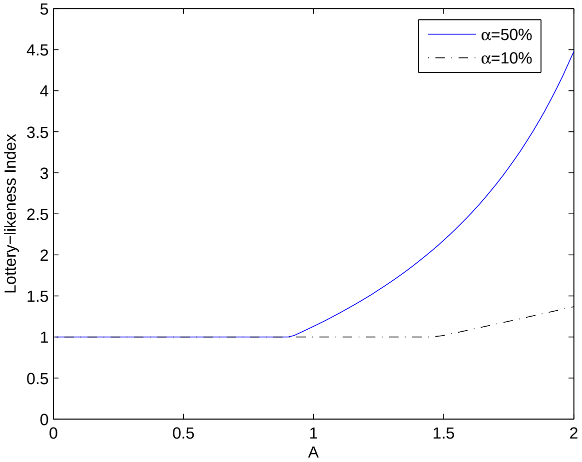
- Initial budget $x_0 = 1$

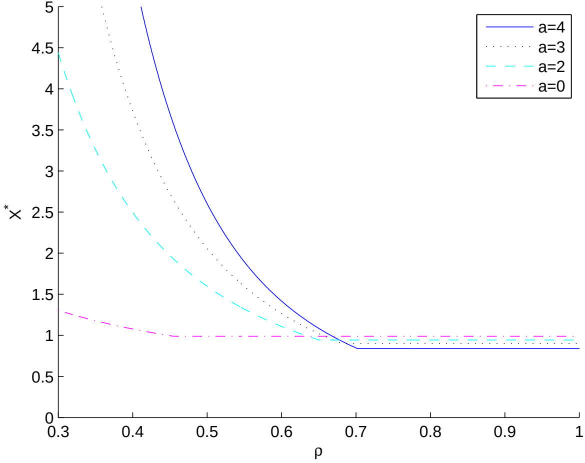
Optimal Solution with Different Aspiration Levels



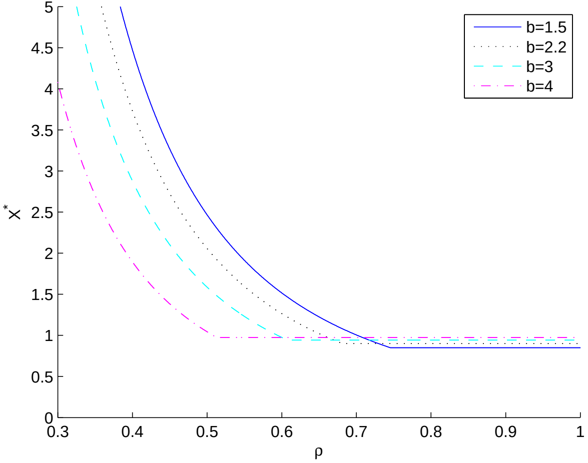
Lottery-likeness Index with Different Confidence Levels



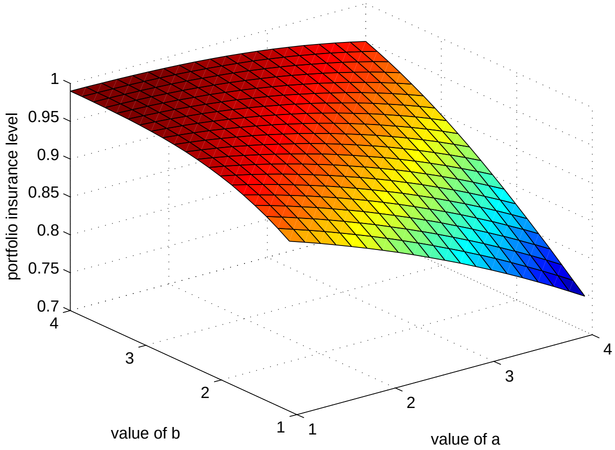
Optimal Solutions with Different Levels of Hope



Optimal Solutions with Different Levels of Fear



Portfolio Insurance Level



Dynamic Portfolio

- Optimal dynamic portfolio (the dollar amount in stock at each time t)

$$\pi(t) = \Delta(t, \rho(t)) \frac{1}{\eta} (\sigma^\top)^{-1} \theta X(t), \quad 0 \le t < T.$$

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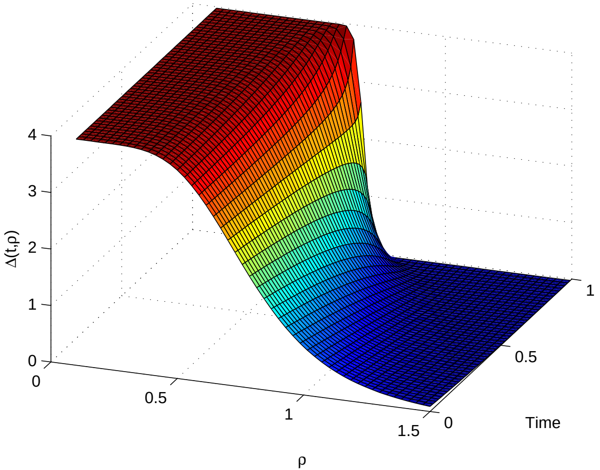
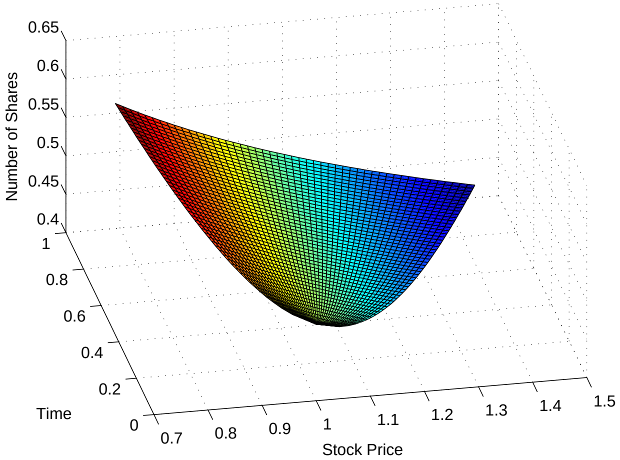
- $\Delta(t, \rho)$ is strictly increasing w.r.t ρ and

$$\lim_{\rho \downarrow 0} \Delta(t, \rho) = a + 1, \quad \lim_{\rho \uparrow \infty} \Delta(t, \rho) = 0$$

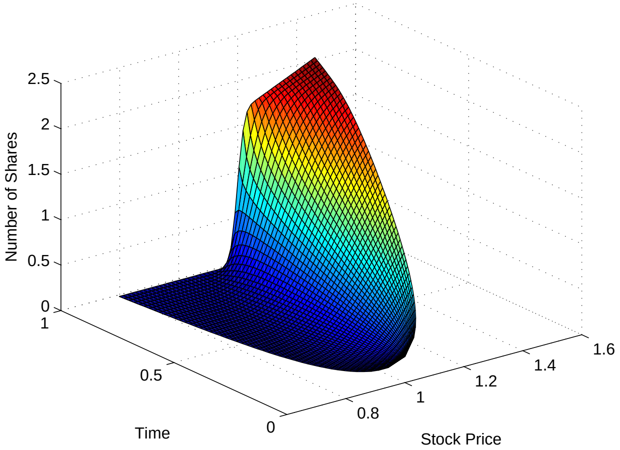
Deviation from EUT



Number of Shares in Stock: EUT with $\eta = 5$



Number of Shares in Stock: HF/A with $\eta = 5$



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- Strong fear leads to portfolio insurance
- Hope makes investors gamble more on good situations
- High aspiration produces aggressive strategies