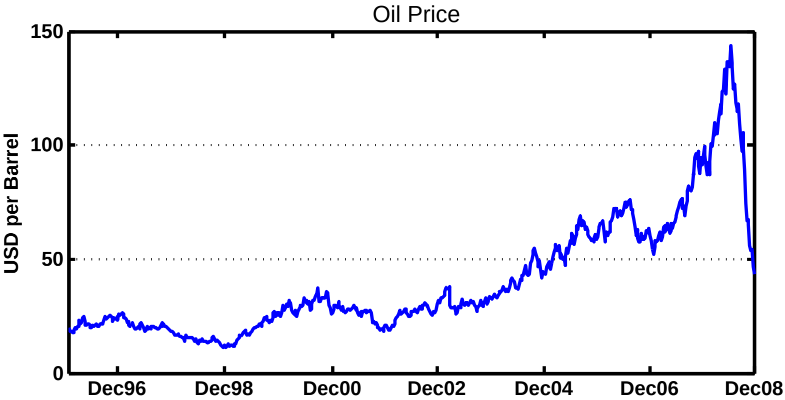
Commodity Derivatives Valuation with Autoregressive and Moving Average Components in the Price Dynamics

Marcel Prokopczuk
ICMA Centre - Henley Business School

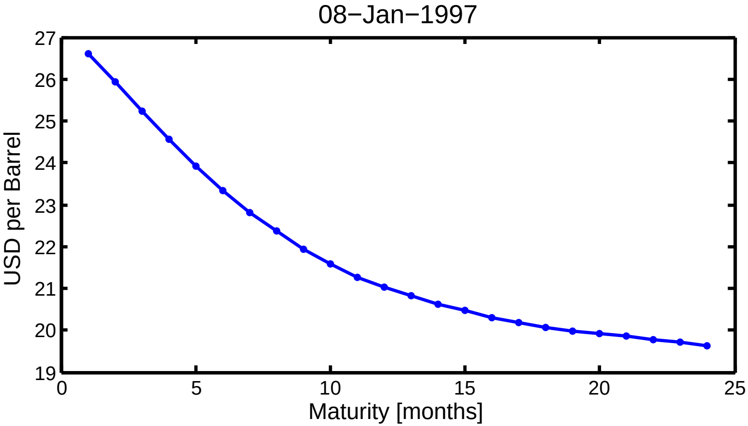
joint work with
Raphael Paschke, Munich Re

Bachelier Congress, Toronto, 2010

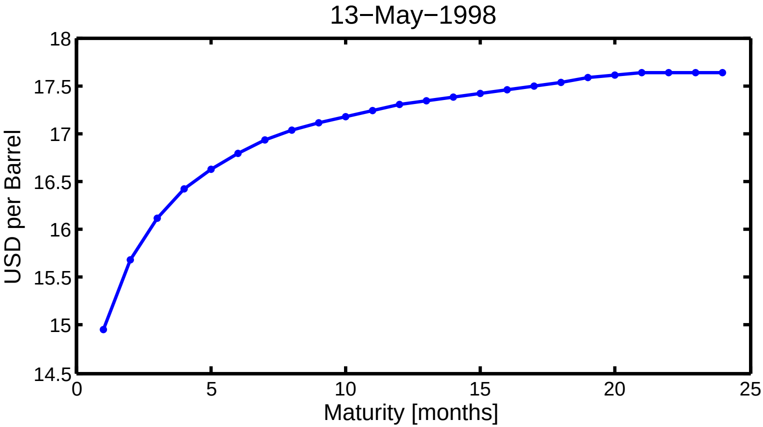
Motivation: Oil Price Development



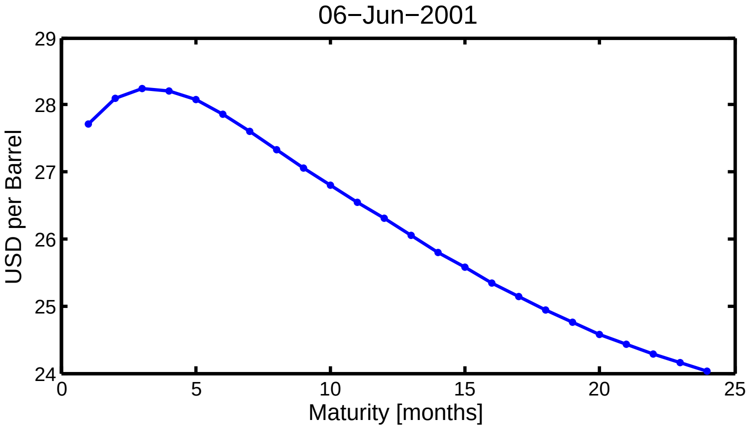
Motivation: Term Structure of Futures Prices



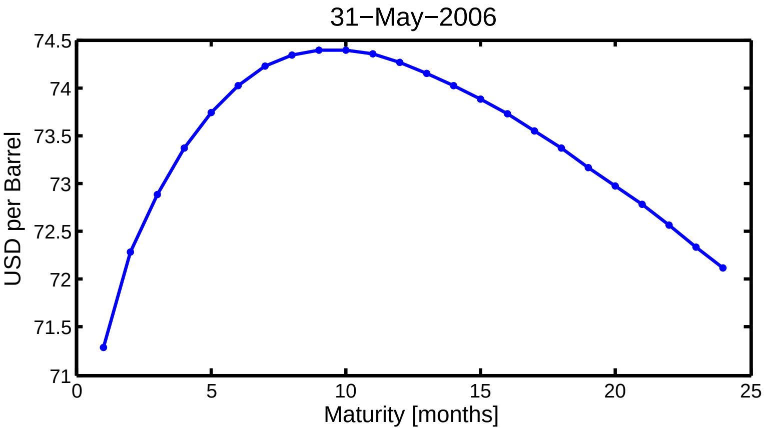
Motivation: Term Structure of Futures Prices



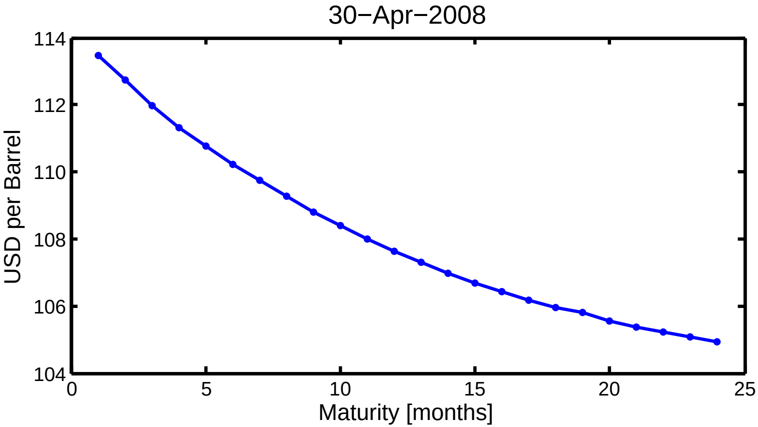
Motivation: Term Structure of Futures Prices



Motivation: Term Structure of Futures Prices



Motivation: Term Structure of Futures Prices



Pricing of Futures Contracts

Futures contracts can be priced by no-arbitrage arguments:

- Financial contracts: Cost-of-carry

$$F_t(T) = S_t e^{r(T-t)}$$

- With storage costs:

$$F_t(T) = S_t e^{(r+s)(T-t)}$$

- No explanation for backwardation
- Inferior empirical performance
- Equity futures can show backwardation due to dividends

$$F_t(T) = S_t e^{(r-q)(T-t)}$$

Pricing of Commodity Futures Contracts

- Commodities differ from pure financial assets as they are hold for consumption or production
- Similar to the dividend yield of a stock, the holder of the commodity receives a **convenience yield** from holding stocks of commodities
- Kaldor (1939): *"... stocks of goods... also have a yield..., by enabling the producer to lay hands on them the moment they are wanted, and thus saving the cost and trouble of ordering frequent deliveries, or waiting for deliveries."*

The Convenience Yield

Convenience yield deterministic function of price:

- Brennan/Schwartz (1985)
- Brennan (1991)

➔ Poor empirical performance

Stochastic convenience yield:

- Gibson/Schwartz (1990)
- Schwartz (1997)
- Schwartz/Smith (2000)
- Cassasus/Collin-Dufresne (2005)

➔ All models assume explicitly or implicitly that the **convenience yield** follows an **Ornstein-Uhlenbeck process**

Modelling the Convenience Yield

- The assumed **Ornstein-Uhlenbeck process** is the **continuous limit of an AR(1)** process

- An analysis of the approximated (net) convenience yield

$$\delta_{t,T-1,T} = \ln\left(\frac{F(t,T)}{F(t,T-1)}\right)$$

  shows that an **AR(1) is not able to capture the dynamics appropriately**

- An **ARMA(1,1)** or higher order AR(q) model yield much **better fit to the data**

Modelling Idea

- Model the convenience yield as continuous autoregressive moving average process: **CARMA(p,q)**
- CARMA(p,q) processes have a long history in the statistics literature: Doob (1944), ..., Brockwell (2001)
- No usage in the finance literature
- One exception for interest rates:
  Benth, Koekebakker, and Zakamouline (2008)

Contribution

Our contribution to the literature:

1. Formulation of a **commodity pricing model** in continuous time allowing for **higher order auto-regression** and **moving average** components:

   ABM-CARMA(p,q)

2. Derivation of **closed-form solutions** for **futures** and **options prices**

3. Application to the **crude oil futures market**, demonstrating the model's **superior empirical performance**

Model Description: *ABM-CARMA(2,1)*

Latent factor spot price model in continuous time:

- One non-stationary factor Z_t: **long-term equilibrium** modelled by an Arithmetic Brownian Motion
- One stationary factor Y_t: **short-term deviations** from the equilibrium modelled by a CARMA(1,0) process (Schwartz/Smith 2000)

$$\ln S_t = Z_t + Y_t$$

$$dZ_t = \mu dt + \sigma_Z dW_t^Z$$
$$dY_t = -kY_t dt + \sigma_Y dW_t^Y$$

Model Description: *ABM-CARMA(2,1)*

Latent factor spot price model in continuous time:

- One non-stationary factor Z_t: **long-term equilibrium** modelled by an Arithmetic Brownian Motion
- One stationary factor Y_t: **short-term deviations** from the equilibrium modelled by a CARMA(2,0) process

$$\ln S_t = Z_t + Y_t$$

$$dZ_t = \mu dt + \sigma_Z dW_t^Z$$
$$d\dot{Y}_t = -k\dot{Y}_t dt + \sigma_Y dW_t^Y$$
$$dY_t = \dot{Y}_t dt$$

Model Description: *ABM-CARMA(2,1)*

Latent factor spot price model in continuous time:

- One non-stationary factor Z_t: **long-term equilibrium** modelled by an Arithmetic Brownian Motion
- One stationary factor Y_t: **short-term deviations** from the equilibrium modelled by a CARMA(2,1) process

$$\ln S_t = Z_t + Y_t$$

$$dZ_t = \mu dt + \sigma_Z dW_t^Z$$
$$d\dot{Y}_t = -k\dot{Y}_t dt + \sigma_Y dW_t^Y$$
$$dY_t = \dot{Y}_t dt$$

Model Description: *ABM-CARMA(2,1)*

Latent factor spot price model in continuous time:

- One non-stationary factor Z_t: **long-term equilibrium** modelled by an Arithmetic Brownian Motion
- One stationary factor Y_t: **short-term deviations** from the equilibrium modelled by a CARMA(2,1) process

$$\ln S_t = Z_t + Y_t + \beta \dot{Y}_t$$

$$dZ_t = \mu dt + \sigma_Z dW_t^Z$$
$$d\dot{Y}_t = -k\dot{Y}_t dt + \sigma_Y dW_t^Y$$
$$dY_t = \dot{Y}_t dt$$

Model Discussion

Model is formulated **directly under the equivalent martingale measure**

Closed form (affine) solutions for the **futures price**:

$$\ln F(Y_t, \dot{Y}_t, Z_t, t; T) = \underbrace{Z_t + A}_{\text{ABM}} + \underbrace{B\dot{Y}_t + CY_t + D}_{CARMA}$$

Difference to the standard Schwartz/Smith 2000 model:

- Term structure:

  Much **more flexible**, especially at the **short end**

- Volatilities:

  Non-monotonous structure and **higher curvature**

Model Implementation: Data

Data used:

- **Crude oil future**s traded at the New York Mercantile Exchange (NYMEX)
- Sample period: January 1996 to December 2008
- Weekly observations (Wednesday)
- Maturities 1 to 24 months
- Data source: Bloomberg

➔ **Panel data set of 676 x 24 observations**

Model Implementation: Estimation

Implementation of the **ABM-CARMA(2,1)** model:

- Write discretized version in **state space** form
- Dynamics of latent factors:
  Translation equation
- Add measurement error to the pricing formula:
  Measurement equation
- **Kalman filter maximum likelihood** estimation of parameters

➔ **Benchmark: Schwartz/Smith (2000)**

In-Sample Pricing Errors

	Root Mean Squared Error				
	Absolute		%-Decrease	Relative	
F01	0.0409	0.0486	**15.8%**	1.26%	1.49%
F02	0.0283	0.0330	**14.2%**	0.87%	1.02%
F03	0.0207	0.0230	**10.0%**	0.64%	0.71%
All	0.0122	0.0141	**13.5%**	0.38%	0.43%

$$AIC_{ABM-CARMA} = -157,285, \qquad AIC_{SS2000} = -152,935,$$
$$SIC_{ABM-CARMA} = -157,131, \qquad SIC_{SS2000} = -152,795.$$

Out-of-Sample Pricing Errors: Time-Series

Split Data Sample into two periods

- Estimation: First half
- Prediction: Second half

	Root Mean Squared Error				
	Absolute		%-Decrease	Relative	
F01	0.0564	0.0627	**10.1%**	1.48%	1.59%
F02	0.0510	0.0543	**5.9%**	1.33%	1.38%
F03	0.0472	0.0488	**3.2%**	1.23%	1.24%
All	0.0375	0.0381	**1.6%**	0.94%	0.95%

Out-of-Sample Pricing Errors: Cross-Section

Split Data Sample into two parts

- Estimation: F01 - F12
- Prediction: F13 - F24

	Root Mean Squared Error				
	Absolute		%-Decrease	Relative	
F15	0.0068	0.0090	**24.4%**	0.21%	0.29%
F18	0.0115	0.0149	**22.8%**	0.36%	0.48%
F21	0.0173	0.0216	**19.9%**	0.55%	0.71%
F24	0.0237	0.0284	**16.5%**	0.75%	0.93%
All	0.0144	0.0179	**19.6%**	0.46%	0.59%

Conclusion

- **AR(1) poor description of the convenience yield**
- **Extension** of Schwartz/Smith model using **continuous time limit of ARMA** processes to describe the convenience yield
- Results in:
  - **More flexible futures curves**
  - **Without** the use of **additional risk factors**
- Applied to crude oil futures:
  - **Better fit/prediction at the short end**
  - **Better prediction of long maturity contracts from short maturity contracts**