Overprized options on variance swaps in local vol models

Mathias Beiglböck, joint with Peter Friz and Stephan Sturm

Universität Wien

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Outline

1 Setting:
   - Stochastic Volatility
   - Local Volatility - Gyöngy - Dupire
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2 Nice Conjecture
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   - Stochastic Volatility
   - Local Volatility - Gyöngy - Dupire

2. Nice Conjecture

3. Counterexample
Setting

Assumptions:

1. $r = 0$.
2. fixed Martingale measure $\mathbb{P}$.
3. time horizon: $[0, T]$. 

\[dS_t(\omega) = S_t(\omega) \sigma(t, S_t(\omega)) dB_t(\omega), \quad \sigma = \sigma(t, \cdot) \text{ progressively measurable.}\]

\[d\tilde{S}_t(\omega) = \tilde{S}_t(\omega) \tilde{\sigma}(t, \tilde{S}_t(\omega)) dB_t(\omega), \quad \tilde{\sigma} = \sigma(t, s) \text{ is deterministic.}\]
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stochastic vol model:

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dS_t(\omega) = S_t(\omega)\sigma(t, \omega)dB_t(\omega), \sigma = \sigma(t, \omega)
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\( \sigma(t, \omega) \) progressively measurable.
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local vol:

\[
d\tilde{S}_t(\omega) = \tilde{S}_t(\omega)\tilde{\sigma}(t, \tilde{S}_t(\omega))dB_t(\omega)
\]

\( \sigma = \sigma(t, s) \) is deterministic.
Theorem (Gyöngy, ’86)

Assume $S$ satisfies $dS_t(\omega) = S_t(\omega)\sigma(t, \omega)dB_t(\omega), \sigma = \sigma(t, \omega)$. 

There exists a deterministic $\tilde{\sigma} = \tilde{\sigma}(t, s)$ so that $\tilde{S}$, given by $d\tilde{S}_t = \tilde{S}_t \tilde{\sigma}(t, \tilde{S}_t)dB_t$, satisfies law $(S_t)$ = law $(\tilde{S}_t)$ for all $t \in [0, T]$.

For the explicit representation: $\tilde{\sigma}^2(t, s) = E[\sigma^2(t, \omega) | S_t = s]$. 

The price of European call $C = C(t, K)$ depends solely on law $(S_t)$ and $(\tilde{S}_t)$ generates the same call prices $C = C(t, K)$. 

M. Beiglböck (Universität Wien)
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$$d\tilde{S}_t = \tilde{S}_t\tilde{\sigma}(t, \tilde{S}_t) dB_t$$

satisfies $\text{law}(S_t) = \text{law}(\tilde{S}_t)$ for all $t \in [0, T]$. 
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Theorem (Gyöngy, '86)

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**Explicit representation:** $\tilde{\sigma}^2(t,s) = \mathbb{E}[\sigma^2(t,\omega)|S_t = s]$.

Price of European call $C = C(t,K)$ depends solely on $\text{law}(S_t)$. $\implies (S_t)$ and $(\tilde{S}_t)$ generate the same call prices $C = C(t,K)$. 
Dupire’s formula:

Assume that for $s > 0$, $t \in [0, T]$ call prices $C(t, K)$ are known. Define

$$\tilde{\sigma}^2(t, s) = 2 \frac{\partial_t C(t, s)}{s^2 \partial_{KK} C(t, s)}.$$ 

Then $\tilde{S}$, $d\tilde{S}_t = \tilde{S}_t \tilde{\sigma}(t, \tilde{S}_t) dB_t$ reproduces $C(t, K)$. 
Tempting: Given call prices from the market \((dS = \sigma S dB)\), set up the local vol model, use it to price more complicated options.

Question: useful information for the price of exotic options? we, today: realized variance and options thereon

\[ V = \int_0^T \sigma^2(t) dt \]

resp. \[ \tilde{V} = \int_0^T \tilde{\sigma}^2(t, \tilde{S}(t)) dt \]

Important observation:
\[ E[\tilde{V}] = E[V]. \]

I.e. the variance swap has the same price in stochastic / local vol model:
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E[\tilde{V}] = \int_0^T E \left[ E \left[ \sigma^2(t, S_t = s) | s = \tilde{S}_t \right] \right] dt
\]

\[
= \int_0^T E \left[ E \left[ \sigma^2(t, S_t = s) | s = S_t \right] \right] dt = \int_0^T E[\sigma^2(t)] \, dt = E[V].
\]
by known prices of European options.

Returning to the lower bound, it has been conjectured\footnote{M. Beiglböck (Universität Wien) Overprized options in local vol models June 2010 7 / 15} that the minimum possible value of an option on variance is the one generated from a local volatility model fitted to the volatility surface. Clearly options on variance have value even in a local volatility model because realized variance depends on the realized path of the stock price from inception to expiration. Given that local variance is a risk-neutral conditional expectation of instantaneous variance, it seems obvious that any other model would generate extra fluctuations of the local volatility surface relative to its initial state.

However, their model-independent upper and lower bounds is now
Recall: \[ V = \int_0^T \sigma^2(t) \, dt \quad \tilde{V} = \int_0^T \tilde{\sigma}^2(t, \tilde{S}(t)) \, dt \]

Conjecture: \[ \mathbb{E}[(V - K)^+] \geq \mathbb{E}[(\tilde{V} - K)^+] \quad \text{for all } K > 0. \]
Recall: \( V = \int_0^T \sigma^2(t) \, dt \quad \tilde{V} = \int_0^T \tilde{\sigma}^2(t, \tilde{S}(t)) \, dt \)

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Excursion: convex – order

\( \mu, \tilde{\mu} \) prob. measures on \( \mathbb{R} \),
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$\mu \preceq_c \tilde{\mu}$ : $\iff$

$\int \varphi(x) \, d\mu(x) \geq \int \varphi(x) \, d\tilde{\mu}(x)$ for every convex $\varphi : \mathbb{R} \to \mathbb{R}$.
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Tfae:

- \( \mathbb{E}[(V - K)^+] \geq \mathbb{E}[(\tilde{V} - K)^+] \) for all \( K > 0 \).
- \( \mathbb{E}[\varphi(V)] \geq \mathbb{E}[\varphi(\tilde{V})] \) for every convex \( \varphi : \mathbb{R} \to \mathbb{R} \).
- \( \text{law}(V) \succ_{c} \text{law}(\tilde{V}) \) in the convex order.
Counterexample

Idea: pick a model such that $V$ is $\succeq_c$-minimal, i.e. deterministic.
Counterexample

Idea: pick a model such that $V$ is $\gtrapprox_c$-minimal, i.e. deterministic.

Example: Black–Scholes “mixing” model on $[0, 3]$

\[
\begin{align*}
S_t &= S_0 \exp \left( \int_0^t \sigma_s \, dB_s \right), \\
\sigma_t^2 &= \begin{cases} 
2 & \text{if } t \in [0, 1], \\
1 & \text{if } t \in [1, 2], \\
3 & \text{if } t \in [2, 3].
\end{cases}
\end{align*}
\]

$\mathbb{E} = \Rightarrow V = \int_0^3 \sigma_t^2 \, dt \equiv 6.$
Counterexample

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Example: Black–Scholes “mixing” model on $[0, 3]$

$$dS_t = S_t \sigma_t dB_t, \quad S_0 = 1.$$
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**Example: Black–Scholes “mixing” model on $[0, 3]$**

$$dS_t = S_t \sigma_t dB_t, \quad S_0 = 1.$$

Fair coin flip $\epsilon = \pm 1$ (independent of $B$), $\sigma^2 = \sigma_\epsilon^2$,

$$\sigma^2_+(t) := \begin{cases} 2 & \text{if } t \in [0, 1], \\ 3 & \text{if } t \in ]1, 2], \\ 1 & \text{if } t \in ]2, 3], \end{cases} \quad \sigma^2_-(t) := \begin{cases} 2 & \text{if } t \in [0, 1], \\ 1 & \text{if } t \in ]1, 2], \\ 3 & \text{if } t \in ]2, 3]. \end{cases}$$

$$\int_0^3 \sigma^2_t(t) dt \equiv 6.$$
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    \end{cases}
\]

\[\implies V = \int_0^3 \sigma^2(t) \, dt \equiv 6.\]
Counterexample / local vol part: \( d\tilde{S}_t = \tilde{S}_t \tilde{\sigma}(t, \tilde{S}_t) dB_t \)

\[
\tilde{V} = \int_0^3 \tilde{\sigma}^2(t, \tilde{S}_t) \, dt \text{ is not deterministic:}
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(a) \( \tilde{\sigma}^2(t, s) = \mathbb{E}[\sigma^2(t)|S_t = s] \)
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(a) $\tilde{\sigma}^2(t, s) = \mathbb{E}[\sigma^2(t) | S_t = s]$

(b) $(\tilde{S}_t)$ has full support.
Counterexample / local vol part: \( d\tilde{S}_t = \tilde{S}_t \tilde{\sigma}(t, \tilde{S}_t) dB_t \)

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for yellow paths: \( \int_0^3 \tilde{\sigma}^2(t, \tilde{S}_t(\omega)) \, dt > 6 \)
Counterexample

\[ V = \int_0^3 \sigma(t, \tilde{S}_t) \, dt \equiv 6, \text{ but} \]

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More specific, consider call with strike 6, i.e. \( f(v) := (v - 6)^+ : \)
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More specific, consider call with strike 6, i.e. $f(v) := (v - 6)^+:$

$$\mathbb{E}[(V - 6)^+] = 0$$
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More specific, consider call with strike 6, i.e. \( f(v) := (v - 6)^+ \):

\[ \mathbb{E}[(V - 6)^+] = 0 < \mathbb{E}[(\tilde{V} - 6)^+] \]
Some remarks/variations

1. $\epsilon$ can be chosen adapted to $\sigma(\xi(B_t))_0 \leq t \leq 3 = \Rightarrow$ generalized Black-Scholes-model, in particular complete.

2. $\sigma(., \omega)$ can be chosen in a continuous/smooth way.

3. Using Gyöngy's result in two dimensions, one obtains a counterexample of (time-inhomogenous) Markovian structure.
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Numerically there is some evidence in favor of $V \gtrapprox c \tilde{V}$:

- experiments by Hans Bühler in the Heston-model
- in the above example we find $E[(V - 6)^+] = 0.026$.

Further assumptions are necessary to rigorously prove $V \gtrapprox c \tilde{V}$. 

M. Beiglböck (Universität Wien) 
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\[ \mathbb{E}[(V - 6)^+] = 0, \quad \mathbb{E}[(\tilde{V} - 6)^+] \approx 0.026. \]
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