Contagion and Confusion in Credit Markets

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What is Contagion?

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- Qualitatively, we will say that there is contagion from market $X$ (or time series $X$) to another market $Y$ (or time series $Y$) if $X$ and $Y$ are *more dependent* during times of crisis than during normal, calmer times.
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▶ Question: How do we measure dependence between two time series?
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- $\rho$ is constant.
Linear Models in Finance

Pearson’s $\rho$ is especially suitable for linear factor models in finance, i.e., linear regression models.
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- $\epsilon_t$ is a sequence of independent, identically distributed, centered Gaussian random variables with variance $\sigma^2$
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Example: $Y_t = \alpha + \beta X_t + \epsilon_t$, where

- $\alpha$ and $\beta$ are constants
- $\epsilon_t$ is a sequence of independent, identically distributed, centered Gaussian random variables with variance $\sigma^2$
- $X_t$ is, for example, the excess returns of the market (S&P 500)
- $Y_t$ is, for example, the returns of Caterpillar stock
Contagion or Confusion?
Contagion or Confusion?
Let
\[ m(x) := \mathbb{E}(Y | X = x) = \alpha + \beta x \] (1)
with regression slope \( m'(x) = \beta \).
Extending the Linear Model

Let

\[ m(x) := \mathbb{E}(Y|X = x) = \alpha + \beta x \quad (1) \]

with regression slope \( m'(x) = \beta \). It also follows that the regression slope \( \beta = \rho\sigma_Y / \sigma_X \) and therefore that

\[ \rho = \beta \sigma_X / \sigma_Y . \quad (2) \]
From linear regression theory, we know that we can write the variance $\sigma_Y^2$ of $Y$ as a sum of the variance explained by the regression (namely, $\beta^2 \sigma_X^2$) and the residual (unexplained) variance $\sigma^2$.\[ \sigma_Y^2 = \beta^2 \sigma_X^2 + \sigma^2 \] And hence \[ \rho = \frac{\beta^2 \sigma_X}{\sigma^2 \sqrt{\beta^2 \sigma_X^2 + \sigma^2}} \frac{1}{\sqrt{2}}.\]
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$$\sigma^2_Y = \beta^2 \sigma^2_X + \sigma^2$$  \hspace{1cm} (3)

and hence

$$\rho = \sigma_X \beta / (\sigma^2_X \beta^2 + \sigma^2)^{1/2}.$$  \hspace{1cm} (4)
Extending the Linear Model

We now extend the usual linear regression model

\[ Y_t = \alpha + \beta X_t + \epsilon_t \]  \hspace{1cm} (5)
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and the usual correlation coefficient to

\[ \rho(x) = \sigma_X \beta(x) / (\sigma_X^2 \beta(x)^2 + \sigma^2(x))^{1/2} \]  \hspace{1cm} (7)

where \( m \) and \( \sigma \) are smooth real-valued functions.
We call $\rho$ the *local correlation function*:

$$
\rho(x) = \frac{\sigma_X \beta(x)}{(\sigma_X^2 \beta(x)^2 + \sigma^2(x))^{1/2}}.
$$

(8)
We call $\rho$ the *local correlation function*:

$$\rho(x) = \frac{\sigma_X \beta(x)}{\sigma_X^2 \beta(x)^2 + \sigma^2(x))^{1/2}}. \quad (8)$$

- $\sigma_X$ denotes the unconditional standard deviation of $X$
- $\beta(x) = m'(x)$ is the slope of the regression function $m(x)$
- $\sigma^2(x) = \text{Var}(Y|X = x)$ is the scedastic function
A Spatial Definition of Contagion

Let

- $X_t$ be U.S. stock market returns
- $Y_t$ be French stock market returns
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Moreover, let

- $x_L = F_X^{-1}(0.025)$ be a lower quantile of $X$; and
- $x_M = F_X^{-1}(0.50)$ be a median quantile of $X$. 
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Moreover, let

- $x_L = F_X^{-1}(0.025)$ be a lower quantile of $X$; and
- $x_M = F_X^{-1}(0.50)$ be a median quantile of $X$.

Then we say that there is 	extit{contagion from $X$ to $Y$} if $\rho(x_L) > \rho(x_M)$. 
Developing the Hypothesis Test

We state the relevant hypothesis test:

\[ H_0: \rho(x_L) \leq \rho(x_M) \text{ (no contagion)} \]
\[ H_1: \rho(x_L) > \rho(x_M) \text{ (contagion)}. \]
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which is facilitated by the fact that, under certain limiting conditions,

\[ \hat{\rho}(x) \overset{D}{\to} \mathcal{N}(\rho(x), \hat{\sigma}_{\hat{\rho}(x)}). \]  \hspace{1cm} (9)
Additionally, $\hat{\rho}(x_M)$ and $\hat{\rho}(x_L)$ are asymptotically independent, so long as $x_M \neq x_L$. 
Developing the Hypothesis Test

- Additionally, \( \hat{\rho}(x_M) \) and \( \hat{\rho}(x_L) \) are asymptotically independent, so long as \( x_M \neq x_L \).

- We obtain, by approximating \( \hat{\sigma}^2 \hat{\rho}(x_M) \) and \( \hat{\sigma}^2 \hat{\rho}(x_L) \), a Studentized test statistic:

\[
Z = \frac{\hat{\rho}(x_L) - \hat{\rho}(x_M)}{\sqrt{\hat{\sigma}^2 \hat{\rho}(x_L) + \hat{\sigma}^2 \hat{\rho}(x_M)}} \tag{10}
\]
Take $X_t$ and $Y_t$ to be U.S. and French stock market returns, respectively.
What Might Confusion Be?

Let $x_M = F_X^{-1}(0.50)$ be a median quantile of $X$ and let $x_T$ be a tail quantile of $X_t$ associated with crisis.
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Let $x_M = F_X^{-1}(0.50)$ be a median quantile of $X$ and let $x_T$ be a tail quantile of $X_t$ associated with crisis.

We say there is *confusion* from $X$ to $Y$ if

1. $\rho(x_M) > \rho(x_T)$ and
2. $\rho(x_T) = 0.$
Intuition for Confusion

\[ \rho(x) \]

\[ x_M \]

\[ x_U \]

A Definition of Confusion

Example: U.S. and French Equity Markets

Contagion or Confusion?
A Hypothesis Test For Confusion?

- We can execute the hypothesis test

\[ H_0: \rho(x_T) \geq \rho(x_M) \]
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and, separately, determine if a 95% confidence interval around \( \hat{\rho}(x_T) \) includes the origin.
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- We call this approach the *asymptotic approach*, because it uses the asymptotic behavior of \( \hat{\rho}(x) \).
A Minor Dependence Problem

The events

\[
\{ \omega \in \Omega : \hat{\rho}(x_M) > \hat{\rho}(x_T) \}
\]

(11)
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\[ \{ \omega \in \Omega : \hat{\rho}(x_M) > \hat{\rho}(x_T) \} \]  \hspace{1cm} (11)

and

\[ \{ \omega \in \Omega : 0 \in \left( \hat{\rho}(x_T) - 1.96\hat{\sigma}_{\hat{\rho}(x_T)}, \hat{\rho}(x_T) + 1.96\hat{\sigma}_{\hat{\rho}(x_T)} \right) \} \]  \hspace{1cm} (12)
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are dependent.
A Bootstrapping Approach

We take the raw data and create a bootstrap of the data by resampling from the data with replacement \( n \) times.
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- We do this \( N \) times. Denote the set of bootstraps by \( \{B_1, B_2, \ldots, B_N\} \), where

\[
B_i = \{(X_{i,1}, Y_{i,1}), (X_{i,1}, Y_{i,1}), \ldots, (X_{i,n}, Y_{i,n})\}. \quad (13)
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\[
\hat{\rho}_i(x_M), \hat{\rho}_i(x_T), \hat{\sigma}_i, \hat{\rho}(x_M), \hat{\sigma}_i, \hat{\rho}(x_T) \quad (14)
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\]

- For each bootstrap \( B_i \), we ultimately generate estimates

\[
\left( \hat{\rho}_i(x_M), \hat{\rho}_i(x_T), \hat{\sigma}_i, \hat{\rho}(x_M), \hat{\sigma}_i, \hat{\rho}(x_T) \right) \tag{14}
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A Bootstrapping Approach

We count, over all $N$ bootstraps, the number of times in which

$$\hat{\rho}_i(x_M) - 1.96\hat{\sigma}_i(x_M) > \hat{\rho}_i(x_T) + 1.96\hat{\sigma}_i(x_T)$$  \hspace{1cm} (15)

and

$$\hat{\rho}_i(x_M) - 1.96\hat{\sigma}_i(x_M) < \hat{\rho}_i(x_T) - 1.96\hat{\sigma}_i(x_T).$$  \hspace{1cm} (16)
We count, over all $N$ bootstraps, the number of times in which

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(15)

and

$$\hat{\rho}_i(x_T) - 1.96\hat{\sigma}_i,\hat{\rho}(x_T) < 0 < \hat{\rho}_i(x_T) + 1.96\hat{\sigma}_i,\hat{\rho}(x_T).$$

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(16)

We call the proportion of bootstraps satisfying these two conditions an empirical estimate of the *probability of confusion*. 
7 Years of Credit Default Swap History

Historical credit default swap premia for Bear Stearns, Ambac, Citigroup, J.P. Morgan Chase, and Freddie Mac.
Results

Covariate $X$ is the daily percentage change in Bears Stearns CDS.

<table>
<thead>
<tr>
<th>Dependent</th>
<th>$\hat{\rho}(x_M)$</th>
<th>$\hat{\rho}(x_U)$</th>
<th>$\sigma_{\hat{\rho}(x_M)}$</th>
<th>$\sigma_{\hat{\rho}(x_U)}$</th>
<th>$Z_{\hat{\rho}(x_U) - \hat{\rho}(x_M)}$</th>
<th>$P(\text{Confusion})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deutsche Bank (Subordinated)</td>
<td>0.3438</td>
<td>0.2744</td>
<td>0.0378</td>
<td>0.0942</td>
<td>-0.6832</td>
<td>0.005</td>
</tr>
<tr>
<td>J.P. Morgan Chase</td>
<td>0.6880</td>
<td>0.5382</td>
<td>0.0213</td>
<td>0.0802</td>
<td>-1.8040</td>
<td>0.059</td>
</tr>
<tr>
<td>Fannie Mae</td>
<td>0.4147</td>
<td>0.3044</td>
<td>0.0396</td>
<td>0.1037</td>
<td>-0.9934</td>
<td>0.078</td>
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<tr>
<td>Freddie Mac</td>
<td>0.3978</td>
<td>0.2671</td>
<td>0.0406</td>
<td>0.1075</td>
<td>-1.1375</td>
<td>0.099</td>
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<tr>
<td>Countrywide</td>
<td>0.5956</td>
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<td>0.0259</td>
<td>0.0858</td>
<td>-2.0314*</td>
<td>0.002</td>
</tr>
<tr>
<td>Bank of America</td>
<td>0.5794</td>
<td>0.3793</td>
<td>0.0296</td>
<td>0.1017</td>
<td>-1.8885</td>
<td>0.009</td>
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<tr>
<td>Ambac Assurance</td>
<td>0.3628</td>
<td>0.3900</td>
<td>0.0400</td>
<td>0.0880</td>
<td>0.2818</td>
<td>0.000</td>
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<tr>
<td>Ambac Financial Group</td>
<td>0.3709</td>
<td>0.2797</td>
<td>0.0401</td>
<td>0.1008</td>
<td>-0.8413</td>
<td>0.095</td>
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<tr>
<td>Lehman Brothers</td>
<td>0.8731</td>
<td>0.7204</td>
<td>0.0074</td>
<td>0.0583</td>
<td>-2.5981*</td>
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<tr>
<td>Citigroup</td>
<td>0.5797</td>
<td>0.4260</td>
<td>0.0296</td>
<td>0.0955</td>
<td>-1.5372</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Confusion from Countrywide CDS to Ambac CDS?

Credit Default Swap Premia

J. Hamrick

Contagion or Confusion?
Conclusions

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- There is limited evidence of a condition stronger than the absence of contagion, which we call *confusion*.

- Diversified bond and fixed-income derivative investors do not have to worry about “all correlations going to one” during crises.
References

