Hidden Liquidity and the Optimal Display of Iceberg Orders

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Outline

• Hidden Liquidity and Icebergs in Electronic Exchanges

• How much should the trader hide?
  • The Model
  • Model Input: Hidden Liquidity Statistics
  • Calibration
Limit Order Books

- Almost all electronic exchanges are based on Limit Order Books (LOBs)
  - **Market Orders**: immediate execution
  - **Limit Orders**: stored in the LOB

- Orders are executed according to a set of **Priority Rules**:
  - Price Priority
  - Display Priority
  - Time Priority

- Large orders (limit or market) **move the market**

- Orders may be shielded from public view (Hidden Liquidity)
The Displayed Limit Order Book

State of the Order Book

Price Levels
JAVA, 12/11/2008, 11.04 pm
The „True“ Order Book

State of the Order Book

Hidden Liquidity

Number of Shares

Price Levels
JAVA, 12/11/2008, 11.04 pm
Significance of Hidden Liquidity
How much is hidden?

Europe:
proportion of posted hidden liquidity can take up to 40-50%
Some Statistical Properties of Hidden Liquidity

- Correlation of hidden liquidity ratio in the Spread with:
  - Average Spread: 0.859
  - Average Price: 0.755
  - Average Daily Trading Volume (ADV): -0.212
  - Average Trade Size: -0.322

- HL ratio well explained by average spread ($R^2 > 0.7$):

$$H_{Ratio} = -0.04 + 0.09 \text{ Spread}$$
Typical Example: The Iceberg Order

- Only a fraction of the order is **openly displayed** in the LOB
- The hidden part **loses time priority** over the displayed part

- **How much should we display?**
The Model:
Visible Liquidity has Priority over Hidden Liquidity

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The Model

- What is the optimal display size of an iceberg order?

- Model assumptions:
  - Order placed at a single price level (top of book or in spread)
  - Select display size to maximize expected execution volume

- Model Input:
  - Initial LOB Liquidity
  - Order (market and limit) arrival volumes
  - Hidden Liquidity
The Model:
Optimal Display Curves

- Market Impact Model
  \[ \beta(\Delta) = \beta_0 + \gamma \cdot \Delta^2 \]

- Presence of Hidden Liquidity \( H_j \) "encourages" display

- Market Sensitivity \( \gamma \) "discourages" display
Model Calibration: Forecasting Hidden Liquidity

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Model Calibration:
Obtaining Optimal Display Sizes

Cisco

- $N=30000$ (Shares)
- $T=10$ s
- Top-of-Book Size = 10000 (Shares)

Hewlett-Packard

- $N=5000$ (Shares)
- $T=10$ s
- Top-of-Book Size = 500 (Shares)
Optimal Display Size: The role of Imbalance and Spread

Graph: Optimal Display Size (in Shares)

- Order Size $N = 2000$ Shares

- $0 \leq \text{Spread} < 3$ (cent)
- $3 \leq \text{Spread} < 5$ (cent)
- $5 \leq \text{Spread} < 12$ (cent)

- Relative Imbalance

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Conclusion

• Hidden Liquidity is important feature in LOB-markets

• Statistical properties of HL and forecasting

• Model for Optimal Iceberg Implementation

• Optimal Display Strategies