Real Investments and their Financing: a real options approach

Engelbert Dockner, Jøril Mæland, and Kristian R. Miltersen

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Our Research Question

- Does capital structure decisions (i.e., the financing of the firm) interrelate with (real) investments decisions?
  - Capital structure decisions before the investment
  - The timing of the investment decision itself
  - The financing of the investment
  - Why does capital structure influence investment decisions? (Compare Myers, 1977)
  - Are there situations where it does not influence the real decisions? (Compare Modigliani and Miller, 1958 and 1963)

- How does/should capital structure and investments vary across different industries and competitive settings?
What are we doing?

- Providing a micro foundation to the Dynamic Capital Structure Models
- Making a role for investments
- Analyzing different competitive structures and different industries
  - monopoly, imperfect competition, perfect competition
  - mature versus growth industry
- Discuss covenants and introduce the concept of idealized (optimal?) covenants
Example: a firm with an investment option (real option)

- Current Earnings: $X = 1$
- Current Value: $A = 15$
- Current Capital Structure
  - Equity value: $E = 10$
  - Debt value: $D = 5$
  - Coupon: $c = 0.5$
- The investment option
  - Invest $I = 25$ (i.e. improve production facility)
  - The earnings at each instant in time will be doubled
  - The endogenously determined trigger for when to exercise the option is $X_I = 2$
Example: a firm with an investment option (real option)

- The (optimal) situation just after the investment option is exercised:
  - Earnings: $2X_I = 4$
  - Value: $A_+ = 45$
  - Capital Structure
    - Equity value: $E_+ = 23$
    - Debt value: $D_+ = 22$
    - Coupon: $c_+ = 2.0$

- The situation just before the investment option is exercised:
  - Earnings: $X_I = 2$
  - Value: $A = A_+ - I = 20$
  - Capital Structure
    - Coupon: $c_1 = 0.5$
    - Debt value: $D_1 = 7$
    - Equity value: $E_1 = A - D_1 = 13$
Example: a firm with an investment option (real option)

- How to finance the new investment (at the trigger point, $X_I = 2$)?
  - New debt (assume we approach the same creditors):
    - Coupon of junior debt: $c_2 = c_+ - c_1 = 1.5$
    - Value of junior debt: $D_2 = D_+ - D_1 = 15$
  - New equity: $E_2 = I - D_2 = 10$
- Hence, the situation just after the investment option is exercised:
  - Earnings: $X = 4$
  - Capital Structure
    - Equity value: $E_1 + E_2 = 23$
    - Debt value: $D_1 + D_2 = 22$
    - Coupon: $c_1 + c_2 = 2.0$
  - Value: $A = E_1 + E_2 + D_1 + D_2 = 45$
The Firms Capital Structure Decision

- (Instantaneous) cash flow from the production unit, $\xi_t$
- Firm is financed by
  - Debt with fixed instantaneous coupon rate, $c$, and infinite maturity
  - Equity
- Cash flow to
  - Debt: $(1 - \tau_i)c$
  - Equity: $(1 - \tau_e)(\xi_t - c)$
- An Investor who have invested in both debt and equity:
  $(1 - \tau_e)\xi_t + (\tau_e - \tau_i)c$
- The curse of having debt: Bankruptcy
  - The equity holders have a real option to stop paying the coupons. I.e., if $\xi_t$ becomes too low relative to $c$ the equity holders will exercise this option. Hence, there is a trigger value, $X^B$. (I.e., in terms of the state variable $X$.)


A Firm's Capital Structure

Instantaneous profit

\[ c \]

\[ \xi^B \]

\[ \star \]

Time
A Micro Foundation

- The price of the product at a given quantity demanded, $q$
  \[ p(q) = aX_t^\gamma q^{-\theta} \]
  where $a, \gamma, \theta > 0$ and
  \[ dX_t = X_t\mu dt + X_t\sigma dW_t, X_0 = 1 \]

- Costs of producing a given quantity, $q$
  \[ C(q) = kq^\kappa \]
  Convex costs of producing, i.e. $\kappa > 1$. I.e, decreasing returns of scale

- Profit from producing $q$ units
  \[ qp(q) - C(q) = aX_t^\gamma q^{1-\theta} - kq^\kappa \]
A Micro Foundation

Solutions

- Monopoly: Use market power. i.e., take price impact into account when optimizing over $q$: $q^*_M(X_t)$

- Duopoly: Both competitors take price impact into account
  - Cournot competition: $q^*_C(X_t)$
  - Bertrand competition: $q^*_B(X_t)$

- Perfect Competition: Each producer takes the price, $aX^\kappa_t$, as given, i.e, $\theta = 0$. Hence, profit from producing becomes

$$aX^\gamma_t q - kq^\kappa$$

Therefore, $q^*_P(X_t)$

In all cases will we get instantaneous profit from producing on the form $\xi_t = \omega X^\epsilon_t c^\eta$, $\epsilon > 0$, $\eta < 0$, and $\omega > 0$
The instantaneous profit in the monopoly and perfect competition cases

\[ \xi_t = \omega X_t^\epsilon k^\eta \]

Parameters

\[ \epsilon = \frac{\gamma \kappa}{\kappa + \theta - 1} > 0 \]
\[ \eta = -\frac{1 - \theta}{\kappa + \theta - 1} < 0 \]
\[ \omega = (1 - \theta)^{\frac{1-\theta}{\kappa+\theta-1}} \left( \frac{a}{\kappa} \right)^{\frac{\kappa}{\kappa+\theta-1}} (\kappa + \theta - 1) > 0 \]
Investments and Bankruptcy

- An investment can reduce variable production costs, $k$
  - New approach (as far as we know)
  - Others have looked at capacity constraints: Numerically very complicated
- After a bankruptcy the variable production costs, $k$, may have increased
Different Competitive Settings and Different Industries

- Different types of industries
  - Competitive (low $\theta$) versus non competitive (high $\theta$)
  - Mature (low $\gamma$) versus growth (high $\gamma$)
  - High versus low $\kappa$

![Graph showing different competitive settings and different industries](image)
Combining Investments and Capital Structure

- We have an option to improve the production
  - Invest $I$ at a given date $\tau$
  - After the investment the parameter in the optimal instantaneous cash flow

\[ \xi_t = \omega X_t^\varepsilon k^n \]

changes from unity (1) to $k_f$

- The firm has already some debt in its capital structure with instantaneous coupon, $c$

- (Part of) the capital needed for the new investment, $I$, will be raised by issuing more debt in the firm (with instantaneous coupon, $c_J$)
Combining Investments and Capital Structure

- We have to be careful with covenants of debt and with how to split the firm value in case of bankruptcy between the two classes of debt.
- Typically, a firm has to default on all its debt at the same point in time.
- The decision to make the investment (and how to finance it) will be taken by the equity holders, i.e., maximizing their future cash flow.
Combining Investments and Capital Structure

What is happening?

- Cash flows before the investment
  - Debt: \((1 - \tau_i)c\)
  - Equity: \((1 - \tau_e)(\omega X^e_t - c)\)

- Cash flows after the investment
  - Debt: \((1 - \tau_i)(c + c_J)\)
  - Equity: \((1 - \tau_e)(\omega X^e_t k^\eta_f - c - c_J)\)

The capital raised by issuing the new debt helps the equity holders finance the new investment, \(I\)
Combining Investments and Capital Structure

A couple of (interesting) questions

- Does it delay or accelerate the decision to make the investment that
  - there is already some debt in the firm's original capital structure (delay)
  - that (part of) the investment capital, \( I \), can be raised by issuing new debt? (accelerate)
- Can we separate the effect of the two issues?
- Does it change the *original* decision to issue debt in the firm that the firm has a (valuable) real option investment opportunity? (reduce initial debt for two reasons—(i) bankruptcy kills the investment option (ii) we get a second chance to increase debt)
Initial conditions when debt is issued

\[ D(1) = P \]
\[ E(1) = A - P \]

- \( P \) is the principal of the debt (issue at par)
- \( A \) is the value of the firm (including its real option to invest)
- \( A = E(1) + D(1) \)
The boundary conditions at the bankruptcy trigger level, $B$

\begin{align*}
E(B) &= 0 \\
E'(B) &= 0 \\
D(B) &= (1 - \alpha)A_0 B^\epsilon k^\eta_b
\end{align*}

- $\alpha$ reflects direct bankruptcy costs
- $k^\eta_b$ reflects indirect bankruptcy costs
- $A_0$ is the value of a similar firm optimally financed but without the investment option
The Boundary Conditions at Investment

- The boundary conditions at the investment trigger level, \( F \)

\[
E(F) = E_0(F) + (P_0 F^\epsilon k_f^n - D(F)) - I = A_0 F^\epsilon k_f^n - D(F) - I
\]

\[
E'(F) = \epsilon A_0 F^{\epsilon - 1} k_f^n - D'(F)
\]

- The debt has no value matching condition

- \( P_0 F^\epsilon k_f^n - D(F) \) is the proceeds from issuing a junior loan under idealized (optimal) covenants

- Coupon rate to junior loan \( c_0 F^\epsilon k_f^n - c \)

- Equity holders choice of investment trigger using idealized covenants is identical to a central planner/manager who optimizes total firm value

\[
E(F) + D(F) = A_0 F^\epsilon k_f^n - I
\]

\[
E'(F) + D'(F) = \epsilon A_0 F^{\epsilon - 1} k_f^n
\]
Alternative Boundary Conditions at Investment

- The boundary conditions at investment with no new debt financing (Myers)
  - The boundary conditions at the investment trigger level, $F$
    \[
    E(F) = E_0^c(F) - I \\
    E'(F) = E_0^{c'}(F) \\
    D(F) = D_0^c(F)
    \]
    - $E_0^c$ and $D_0^c$ denotes values with the same $c$ as chosen initially.
  - The boundary condition at refinancing
    - The boundary conditions at the trigger level, $F$, chosen exogenously
      \[
      E(F) = E_0(F) + \left( P_0 F^c k^{\eta}_f - D(F) \right) = AF^c k^{\eta}_f - D(F)
      \]
    - We pick the same $F$ as for the investment case with idealized covenants
Some Numbers

- Short term (after-tax) interest rate $r = 0.05$
- Volatility on the $X$ process $\sigma = 0.3$
- Drift of the $X$ process $\mu = 0.02$
- Price elasticity of demand $\theta = 0.4$
- Income elasticity of demand $\gamma = 0.5$
- Convexity of cost function $\kappa = 1.2$
- Direct bankruptcy cost $\alpha = 0.2$
- Indirect bankruptcy costs $k_b = 1.2$
- The effective tax rate on dividends $\tau_e = 0.42$
- The tax rate on coupon payments $\tau_i = 0.34$
- Improvement from investment $k_f = 0.6$
- Investment costs $I = 5$
### Some numbers

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<th>Pure E</th>
<th>E&amp;D</th>
<th>E&amp;D</th>
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Some Empirical Implications

- (Initial) leverage ratios depend on
  - Industry: More growth, less leverage
  - Competitiveness: More competitive, less leverage
  - Convexity of costs: ambiguous
  - Moneyness of real investment option(s): more in-the-money, less leverage
- Investment triggers
- Bankruptcy triggers
- In order to implement first best decisions of investments a rich menu of debt covenants to pick from is essential in designing debt contracts
Why are we doing this?

- Investment (and bankruptcy) behavior and the competitive environment
- How does capital structure influence investment decisions (Compare Myers, 1977)
- How taxes influence investments (and bankruptcy) across different industries
- Analyzing bankruptcy treatment
- Return requirements for different types of (optimal) financing of investments
- A rigorous treatment of Weighted Average Cost of Capital (WACC)
- Separation between direct and indirect bankruptcy costs
- Only one investment option per firm (Will be lost in case of bankruptcy before investment option is exercised)
With Competitive Interactions

- So far we have to force firms to have 100% equity financing after bankruptcy
- A leader (who invests first) and a follower
- Preemption for some parameter values
- A new role for debt: To reduce preemption