Own-Company Stockholding and Work Effort Preferences of an Unconstrained Executive

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Center for Mathematical + Computational Modelling

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\(^1\) Joint Work with Alexander Szimayer and John Gould.
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Motivation

- Share-based payments frequently used and controversial; (public interest: Are executives overpaid?)

- Finance and economics theory: principal-agent-problem; (principal = share holder, agent = executive)

- How do share-based payments (e.g.: stock options) increase the executive’s incentive/effort? (“constrained executive”: risk taking in own-company manipulated )

- “Base case” as first step: analyze “unconstrained executive” without any constraints on his compensation.
  ⇒ Insight how the agent can be controlled by the principal.
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Utility-maximizing Executive

- Endowed with an initial wealth $v_0$, which is invested in the money market account, a diversified market portfolio, and own company shares
- Value of his own company is influenced via work effort:
  - Gain in utility from the increased value of his direct shareholding
  - Loss in utility for his work effort $\rightarrow$ disutility term

Characterization of the Executive

- Risk aversion parameter $\gamma$
- Work effectiveness parameters:
  - Inverse work productivity $\kappa$
  - Disutility stress $\alpha$
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Money Market Account:

\[ dB_t = r B_t \, dt, \quad B_0 = 1, \quad (1) \]

Market Portfolio:

\[ dP_t = P_t (\mu^P \, dt + \sigma^P \, dW_t^P), \quad P_0 \in \mathbb{R}^+, \quad (2) \]

Company’s share price process is a controlled diffusion with SDE

\[ dS_t^{\mu,\sigma} = S_t^{\mu,\sigma} \left( \mu_t \, dt + \sigma_t \, dW_t + \beta \left[ \frac{dP_t}{P_t} - r \, dt \right] \right), \quad S_0 \in \mathbb{R}^+, \quad (3) \]

where the drift \( \mu_t \) and the volatility \( \sigma_t \) are controlled by the executive.

---

**Individual influences the own company’s share price.**

\[ \overset{\Delta}{=} \text{Gain in utility from the increased value of his direct shareholding.} \]

**Remark**

\( W^P \) and \( W \) are two independent standard Brownian motions, but the instantaneous correlation between \( S_t^{\mu,\sigma} \) and \( P_t \) is

\[ \rho_t = \frac{\beta \sigma^P}{\sqrt{\sigma^2 + (\beta \sigma^P)}}. \]
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Wealth Equation

For investment strategy $\pi = (\pi^P, \pi^S)$ and initial wealth $V_0 > 0$:

$$dV_t^\pi = V_t^\pi \left( (1 - \pi_t^P - \pi_t^S) dB_t/B_t + \pi_t^P dP_t/P_t + \pi_t^S dS_t^{\mu, \sigma}/S_t^{\mu, \sigma} \right).$$ (4)

Work Effort Choice and Disutility

Instantaneous disutility of work effort is represented by a Markovian disutility rate $c(t, v, \mu_t, \sigma_t)$ for control strategy $(\mu_t, \sigma_t)$.

⇒ The optimal investment and control decision is the solution of

$$\Phi(t, v) = \sup_{(\pi, \mu, \sigma) \in A(t, v)} \mathbb{E}^{t, v} \left[ U(V_T^\pi) - \int_t^T c_u(\mu_u, \sigma_u) \, du \right], \quad (t, v) \in [0, T] \times \mathbb{R}^+. $$ (5)
Wealth Equation

For investment strategy \( \pi = (\pi^P, \pi^S) \) and initial wealth \( V_0 > 0 \):

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dV^\pi_t = V^\pi_t \left( (1 - \pi^P_t - \pi^S_t) dB_t/B_t + \pi^P_t dP_t/P_t + \pi^S_t dS^\mu,\sigma_t/S^\mu,\sigma_t \right).
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Work Effort Choice and Disutility

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\( \Rightarrow \) The \textit{optimal investment and control decision} is the solution of

\[
\Phi(t, \nu) = \sup_{(\pi, \mu, \sigma) \in A(t, \nu)} E^{t, \nu} \left[ U(V^\pi_T) - \int_t^T c_u(\mu_u, \sigma_u) \, du \right], \quad (t, \nu) \in [0, T] \times \mathbb{R}^+.
\] (5)
Define **Sharpe ratio** as \( \lambda = \frac{\mu - r}{\sigma} \).

Minimize disutility rate for this fixed Sharpe ratio \( \lambda \) and obtain \( c^*(t, v, \lambda) \).

Replace \( c(t, v, \mu, \sigma) \) by \( c^*(t, v, \lambda) \).

Restate the maximization problem (5) over the controls \( \pi \) and \( \lambda \).

**Lemma**

*Under sufficient assumptions on \( c(t, v, \mu, \sigma) \), the minimization problem*

\[
\min_{\{\sigma > 0: \mu = r + \lambda \sigma\}} c(t, v, \mu, \sigma), \quad \text{for } (t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+,
\]

*admits a unique solution \( \sigma^*(t, v, \lambda) \).*
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\]

*admits a unique solution \( \sigma^*(t, \nu, \lambda) \).*
Dimension Reduction of the Maximization Problem

**Theorem**

Suppose

\[ \Phi(t, v) = \sup_{(\pi, \mu, \sigma) \in A(t, v)} \mathbb{E}^{t, v} \left[ U(V_T^\pi) - \int_t^T c_u(\mu_u, \sigma_u) \, du \right], \quad (t, v) \in [0, T] \times \mathbb{R}^+ \]

admits a \( C^{1,2} \)-solution \( \Phi \), then it is also the solution of the optimal control problem

\[ \Phi(t, v) = \sup_{(\pi, \lambda) \in A'(t, v)} \mathbb{E}^{t, v} \left[ U(V_T^\pi) - \int_t^T c_u^*(\lambda_u) \, du \right], \quad (t, v) \in [0, T] \times \mathbb{R}^+, \quad (7) \]

where \( c^* \) is defined via

\[ c^*(t, v, \lambda) := c(t, v, r + \lambda \sigma^*(t, v, \lambda), \sigma^*(t, v, \lambda)) = \min_{\{\sigma > 0: \mu = r + \lambda \sigma\}} c(t, v, \mu, \sigma). \quad (8) \]
Dimension Reduction of the Maximization Problem

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admits a $C^{1,2}$-solution $\Phi$, then it is also the solution of the optimal control problem

$$
\Phi(t, v) = \sup_{(\pi, \lambda) \in A'(t, v)} \mathbb{E}^{t,v} \left[ U(V_T^\pi) - \int_t^T c^{*}_u(\lambda_u) \, du \right], \ (t, v) \in [0, T] \times \mathbb{R}^+, \ (7)
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\[ 0 = \sup_{(\pi, \lambda) \in \mathbb{R} \times [0, \infty)} \Phi_t(t, v) + \Phi_v(t, v) v (r + \pi^S \lambda \sigma + [\pi^P + \beta \pi^S](\mu^P - r)) + \frac{1}{2} \Phi_{vv}(t, v) v^2 ([\pi^S \sigma]^2 + [\pi^P \sigma^P + \beta \pi^S \sigma_P]^2) - c^*(t, v, \lambda), \tag{9} \]

where \((t, v) \in [0, T) \times \mathbb{R}^+\), and \(U(v) = \Phi(T, v)\), for \(v \in \mathbb{R}^+\).

\[ \Rightarrow \text{Maximizers } \pi^{P*}, \pi^{S*} \text{ and } \lambda^* \text{ of (9) by establishing the FOCs:} \]

\[ \pi^{P*}(t, v) = -\frac{(\mu^P - r)}{v(\sigma^P)^2} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)} - \beta \pi^{S*}(t, v), \tag{10} \]

\[ \pi^{S*}(t, v) = -\frac{\lambda^*(t, v)}{v \sigma} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)}, \]

where \(\lambda^*\) is the solution of the implicit equation

\[ \lambda \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} + c^*_\lambda(t, v, \lambda) = 0 \quad \text{for all } (t, v) \in [0, T] \times \mathbb{R}^+. \tag{11} \]
\[
0 = \sup_{(\pi, \lambda) \in \mathbb{R} \times [0, \infty)} \Phi_t(t, v) + \Phi_v(t, v) v (r + \pi^S \lambda \sigma + [\pi^P + \beta \pi^S] (\mu^P - r)) \\
+ \frac{1}{2} \Phi_{vv}(t, v) v^2 ([\pi^S \sigma]^2 + [\pi^P \sigma^P + \beta \pi^S \sigma_P]^2) - c^*(t, v, \lambda),
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Substituting the maximizers (10) in the HJB (9) then yields:

\[
\Phi_t(t, v) + \Phi_v(t, v) v r - \frac{1}{2} (\lambda^*)^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} - \frac{1}{2} (\lambda_P)^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} - c^*(t, v, \lambda^*) = 0,
\]

(12)

where \( \lambda_P := \frac{\mu_P - r}{\sigma_P} \).

\[\rightarrow\]

**Goal:**

Solve equation (12) for a special choice of the utility and disutility functions.
Utility and Disutility Functions

The utility function $U$ is assumed to be CRRA, in particular

$$U(v) = \begin{cases} \frac{v^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \text{ --- "Power Utility"} \\ \log(v), & \text{for } \gamma = 1, \text{ --- "Log Utility"} \end{cases}$$

and the minimized disutility $c^*$ satisfies:

$$c^*(t, v, \lambda) = \kappa v^{1-\gamma} \frac{\lambda^\alpha}{\alpha}, \quad \text{for } \gamma > 0,$$

where $\kappa = \text{inverse work productivity}$ and $\alpha = \text{disutility stress}$.

$\Rightarrow$ Characterization of the executive via $\kappa$, $\alpha$ and $\gamma$. 

Sascha Desmettre  The Unconstrained Executive
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The Unconstrained Executive
The Power Utility Case: $\gamma > 0$ and $\gamma \neq 1$

For $\alpha > 2$ and $\gamma \neq 1$ the separation approach

$$\Phi(t, \nu) = f(t) \frac{\nu^{1-\gamma}}{1-\gamma} \quad \text{with} \quad f(T) = 1$$

substituted in PDE (12) produces a Bernoulli ODE (for $n \neq 1$) of the form

$$\dot{f} = a_1 f + a_n f^n.$$ 

The solution is

$$f(t)^{1-n} = C e^{G(t)} + (1-n) e^{G(t)} \int_0^t e^{-G(s)} a_n \, ds,$$

where $G(t) = (1-n) \int_0^t a_1(s) \, ds$, and $C$ is an arbitrary constant.
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where $G(t) = (1-n) \int_0^t a_1(s) \, ds$, and $C$ is an arbitrary constant.
→ Solutions:

\[ \lambda^*(t, \nu) = \left( \frac{1}{\kappa \gamma} f(t) \right)^{\frac{1}{\alpha-2}} \]  

\[ \pi^{P*}(t, \nu) = \frac{\mu^P - r}{\gamma (\sigma P)^2}, \quad \pi^{S*}(t, \nu) = \frac{\lambda^*(t, \nu)}{\gamma \sigma^*(t, \nu, \lambda^*(t, \nu))}, \]  

\[ \Phi(t, \nu) = \frac{\nu^{1-\gamma}}{1-\gamma} f(t), \]  

where

\[ f(t) = e^{(1-\gamma) \left( r + \frac{1}{2} \frac{\lambda^2 P}{\gamma} \right) (T-t)} \left( 1 - \frac{(\alpha - 2) \left( \frac{1}{\kappa \gamma} \right)^{\frac{2}{\alpha-2}}}{\alpha \left( 2 \gamma r + \frac{\lambda^2 P}{2} \right)} \left( e^{\frac{1-\gamma}{\alpha-2} \left( 2 r + \frac{\lambda^2 P}{\gamma} \right) (T-t)} - 1 \right) \right)^{-\frac{\alpha-2}{2}}. \]
The Log Utility Case: $\gamma = 1$

For $\gamma = 1$ (log-utility) the solution $\Phi$ can be derived by assuming an additive structure of the form

$$\Phi(t, v) = \log(v) + \varphi(T - t).$$

→ **Solutions:**

$$\lambda^*(t, v) = \kappa\frac{1}{\alpha - 2}, \quad \pi^P(t, v) = \frac{\mu^P - r}{\sigma^P}^2, \quad \text{and} \quad \pi^S(t, v) = \frac{\lambda^*(t, v)}{\sigma^*(t, v)\lambda^*(t, v)},$$

and value function

$$\Phi(t, v) = \log(v) + \left[ r + \frac{1}{2} \left( \frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\alpha} \kappa\frac{2}{\alpha - 2} \right] (T - t).$$

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Theoretical results are analyzed for practical insights:

- Investigate executive performance $\lambda^*$ for sensitivities!
  (w.r.t.: work productivity $\kappa^{-1}$, disutility stress $\alpha$)

- How much compensation is appropriate?
  (log-utility setting, indifference utility equivalence principle)

Parameters:

- **investments:**
  - risk-free rate: $r = 5\%$;
  - market portfolio: $\mu^P = 7\%$ and $\sigma^P = 20\%$;
  - own company: $\sigma^*(t, v, \lambda^*) = 40\%$;

- **executive:**
  - time horizon: $T = 10$ years;
  - initial wealth $v = \$5$ Mio.;
  - work productivity: $100 \leq \kappa^{-1} \leq 2000$;
  - disutility stress: $4 \leq \alpha \leq 6$;
Theoretical results are analyzed for practical insights:

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Optimal Effort $\lambda^*$ under Log-Utility

**Figure:** The optimal choice of the executive’s effort parameter $\lambda^*$ graphed against $1/\kappa$ and $\alpha$. 
Indifference Utility Approach for the Log-Utility Case

The executive’s utility from his optimal personal investment and work effort decision is:

$$
\Phi(0, v) = \log v + \left[ r + \frac{1}{2} (\lambda^P)^2 + \frac{1}{2} (\lambda^*)^2 \frac{\alpha - 2}{\alpha} \right] T.
$$

An outside investor’s utility who invests optimally in the executive’s portfolio strategy $\pi^*$ (without spending work effort) is:

$$
\hat{\Phi}(0, v) = \log v + \left[ r + \frac{1}{2} (\lambda^P)^2 + \frac{1}{2} (\lambda^*)^2 \right] T.
$$

⇒ Loss of utility: $\Phi(0, v) - \hat{\Phi}(0, v) = -\frac{1}{\alpha} (\lambda^*)^2 T$

⇒ Using the indifference utility argument $\Phi(0, v + \Delta v) = \hat{\Phi}(0, v)$ yields

$$
\Delta v = v \left( e^{\frac{(\lambda^*)^2 T}{\alpha}} - 1 \right) = v \left( e^{\frac{\lambda^2_{0} T}{\alpha} \left( \frac{\lambda^2_{0}}{\kappa} \right) \frac{2 - 2}{\alpha - 2}} - 1 \right).
$$

⇒ Loss of utility is compensated.
Executive’s “Fair” Pay $\Delta v$ under Log-Utility

Figure: The executive’s fair up-front cash compensation $\Delta v$ (based on indifference utility) graphed against $1/\kappa$ and $\alpha$; with initial wealth $v = 5$ Mio. and $T = 10$. 
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Extensions of the “base case”:

- Closed-form solutions exist also for an exponential utility of wealth;
- Include consumption and time preferences (consumption and work effort) in the present model:
  - Log utility case $\gamma = 1$: Closed-form solution preserved.
  - Power utility case $\gamma \neq 1$: Solve an inhomogeneous Bernoulli ODE; works for $\alpha = 2\gamma + 2$.

Towards the “constrained executive”:

- Develop dynamic “game” with company determining executive’s own-company shareholding and executive controlling effort and other investment decision → Modeled as a Stackelberg differential game;
- Determine optimal mixed compensation (cash, shares, and options);
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