Localizing Temperature Risk

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Weather

PricewaterhouseCoopers Survey 2005 releases the Top 5 sectors in need of financial instruments to hedge weather risk.

Figure 1: PwC survey 2005 for Weather Risk Management Association
Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Meteorological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption gain 10% if temperature increases by 3° C
  - If temperature in Chicago is less than 0° C consumption of orange juice declines 10% on average
Examples

- Natural gas company suffers negative impact in a mild winter
- Construction companies buy weather derivatives (rain period)
- Cloth retailers sell fewer clothes in a hot summer
- Salmon fishery suffer losses by increase of sea temperatures
- Ice cream producers hedge against cold summers
- Disney World (rain period)
Motivation

What are Weather Derivatives (WD)?

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Futures/Options
- 24 US, 6 Canadian, 9 European and 3 Asian-Pacific cities
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009
WD market

CME offers weather contracts on 42 cities throughout the world
Weather Derivatives

CME products

- HDD($\tau_1, \tau_2$) = $\int_{\tau_1}^{\tau_2} \max(18^\circ \text{C} - T_t, 0) dt$
- CDD($\tau_1, \tau_2$) = $\int_{\tau_1}^{\tau_2} \max(T_t - 18^\circ \text{C}, 0) dt$
- CAT($\tau_1, \tau_2$) = $\int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,max} + T_{t,min}}{2}$
- AAT($\tau_1, \tau_2$) = $\int_{\tau_1}^{\tau_2} \tilde{T}_t dt$, where $\tilde{T}_t = \frac{1}{24} \int_1^{24} T_{t_i} dt_i$ and $T_{t_i}$ denotes the temperature of hour $t_i$, (also referred to as C24AT index).
Algorithm

\[ T_t \]
\[ \downarrow \]
\[ X_t = T_t - \Lambda_t \]
\[ \downarrow \]
\[ X_{t+p} = a^\top X_t + \sigma_t \varepsilon_t \]
\[ \downarrow \]
\[ \hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim \mathcal{N}(0, 1) \]

**Motivation**

**Algorithm**

**Econometrics**

\[ T_t \]
\[ \downarrow \]
\[ X_t = T_t - \Lambda_t \]

**Fin. Mathematics.**

\[ CAR(3) \]
\[ \downarrow \]
\[ F_{CAT(t,\tau_1,\tau_2)} \]
\[ \downarrow \]
\[ MPR \]

**Localizing Temperature Residual**
How to smooth the seasonal variance curve?
How close are the residuals to $N(0, 1)$?
How to price no CME listed cities?

Figure 2: Daily empirical variance, seasonal variance $\hat{\sigma}_{t, FTSG}^2$, $\hat{\sigma}_{t, LLR}^2$ using Epanechnikov Kernel and bandwidth $h = 4.49$ for Beijing

Localizing Temperature Residual
Outline

1. Motivation ✓
2. Weather Dynamics
3. Stochastic Pricing of WD
4. Local Temperature Risk
Weather Dynamics: Asian Data

Temperature Market (CME): Tokyo and Osaka
AAT Index

Can we make money?

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Table 1: Osaka AAT contracts listed at CME on 20090130. Source: Bloomberg. CME$^1$ prices of AAT Futures as listed on CME, AAT$^2$ AAT index values computed from the realized temperature data.

Localizing Temperature Residual
Temperature: \( T_t = X_t + \Lambda_t \)

Seasonal function with trend:

\[
\Lambda_t = a + bt + \sum_{l=1}^{L} c_l \cos \left\{ \frac{2\pi (t - d_l)}{l \cdot 365} \right\}
\]

(1)

\( \hat{a} \): average temperature, \( \hat{b} \): global Warming.

<table>
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<tr>
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<td>Kaohsiung</td>
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Table 2: Daily average temperature data. Source: Bloomberg, Deutsche Wetter Dienst.
Seasonality

Seasonal function with trend:

\[ \hat{\Lambda}_t = 24.4 + 16 \cdot 10^{-5} t + \sum_{i=1}^{3} \hat{c}_i \cdot \cos \left\{ \frac{2\pi i(t - \hat{d}_i)}{365} \right\} \]

\[ + \mathcal{I}(t \in \omega) \cdot \sum_{i=4}^{6} \hat{c}_i \cdot \cos \left\{ \frac{2\pi (i - 4)(t - \hat{d}_i)}{365} \right\}, \]

with \( \mathcal{I}(t \in \omega) \) an indicator for December, January and February.

Seasonal function with a Local linear Regression (LLNR),
\[ t = 1 \cdots 365 \text{ days:} \]

\[ \arg \min_{e,f} \sum_{t=1}^{365} \left\{ \bar{T}_s - e_s - f_s(t - s) \right\}^2 K \left( \frac{t - s}{h} \right) \] (2)
Figure 3: The Fourier truncated, the corrected Fourier and the local linear seasonal component for daily average temperatures in Tokyo Narita International Airport (upper left), Osaka Kansai International Airport (upper right), Beijing (lower left), Taipei (lower right).
AR(p): $X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t$

Figure 4: Residuals $\hat{\varepsilon}_t$ (left) and squared residuals $\hat{\varepsilon}_t^2$ (right) of the AR(p) (Beijing (up), Taipei (down)). No rejection of $H_0$ that the residuals are uncorrelated at 0% significance level, according to the modified Li-McLeod Portmanteau test.
Seasonal Volatility:

Highly seasonal ACF for squared residuals of AR(p)

Figure 5: ACF for residuals $\hat{\varepsilon}_t$ (left) and squared residuals $\hat{\varepsilon}^2_t$ (right) of the AR(p) for Beijing (up), Taipei (down).
Calibration of Seasonal Variance: $\sigma^2_t$

Calibration of daily variances of residuals AR(3) for 36 years:

- 2 Steps: Fourier truncated series + GARCH(p,q): $\hat{\sigma}^2_{t,FTSG}$

$$\hat{\sigma}^2_{t,FTSG} = c_1 + \sum_{l=1}^{L} \left\{ c_{2l} \cos \left( \frac{2l\pi t}{365} \right) + c_{2l+1} \sin \left( \frac{2l\pi t}{365} \right) \right\} + \alpha_1 (\sigma^2_{t-1}\varepsilon_{t-1})^2 + \beta_1 \sigma^2_{t-1} \quad (3)$$

- 1 Step: Local linear Regression (LLR): $\hat{\sigma}^2_{t,LLR}$

$$\arg\min_{g,h} \sum_{t=1}^{365} \left\{ \hat{\varepsilon}^2_t - g_s - h_s(t-s) \right\}^2 K \left( \frac{t-s}{h} \right) \quad (4)$$
Figure 6: Daily empirical variance, seasonal variance $\hat{\sigma}^2_{t,FTSG}$, $\hat{\sigma}^2_{t,LLR}$ using Epanechnikov Kernel and bandwidth $h = 4.49$ for Tokyo (upper left), Osaka (upper right), Beijing (lower left), Taipei (lower right).
ACF of (Squared) Residuals after Correcting Seasonal Volatility

Figure 7: (Left) Right: ACF for temperature (squared) residuals $\frac{\hat{\epsilon}_t}{\hat{\sigma}_{t,LLR}}$ for Beijing (up), Taipei (down)
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<th>City</th>
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Table 3: Skewness, kurtosis, Jarque Bera (JB), Kolmogorov Smirnov (KS) and Anderson Darling (AD) test statistics (365 days). Critical value at at 5% significance level is 5.99, at 1% – 9.21.
Fitting $\hat{\sigma}_t$: 1-2 Step

Residuals ($\frac{\hat{\epsilon}_t}{\hat{\sigma}_t}$) become normal:

Figure 8: QQ-plot of residuals $\frac{\hat{\epsilon}_t}{\hat{\sigma}_t,FTSG}$ (upper) $\frac{\hat{\epsilon}_t}{\hat{\sigma}_t,LLR}$ (lower) and Normal residuals of Tokyo (upper left), Osaka (upper right), Beijing (lower left), Taipei (lower right)
Temperature Dynamics

Temperature time series:

\[ T_t = \Lambda_t + X_t \]

with seasonal function \( \Lambda_t \). \( X_t \) can be seen as a discretization of a continuous-time process AR(p) (CAR(p)).

This stochastic model allows CAR(p) futures/options pricing.
Stochastic Pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

$$d\mathbf{X}_t = A\mathbf{X}_t dt + e_p \sigma_t dB_t$$

$e_k$: $k$th unit vector in $\mathbb{R}^p$ for $k = 1, \ldots, p$, $\sigma_t > 0$, $A$: $(p \times p)$-matrix

$$A = \begin{pmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & 0 & 1 \\
-\alpha_p & -\alpha_{p-1} & \cdots & -\alpha_1
\end{pmatrix}$$

Go to proof details

Localizing Temperature Residual
Stationarity condition

Solution of $X_t = x \in \mathbb{R}^p$, $s \geq t \geq 0$:

$$X_s = \exp\{A(s - t)\}x + \int_t^s \exp\{A(s - u)\}e_p\sigma_u dB_u$$

is stationarity as long as all the eigenvalues $\lambda_1, \ldots, \lambda_p$ of $A$ have negative real parts, i.e. the variance matrix:

$$\int_0^t \sigma_{t-s}^2 \exp\{A(s)\}e_p e_p^\top \exp\left\{A^\top(s)\right\} ds$$

converges as $t \to \infty$. 

Localizing Temperature Residual
Temperature Futures Price

∃ $Q_\theta$ pricing so that:

$$F_{(t,\tau_1,\tau_2)} = E^{Q_\theta} [Y|\mathcal{F}_t]$$

where $Y$ equals the payoff of the temperature index and by Girsanov theorem:

$$B^\theta_t = B_t - \int_0^t \theta_u \, du$$

is a Brownian motion for $t \leq \tau_{\text{max}}$. $\theta$: a real valued, bounded and piecewise continuous function (market price of risk)
Temperature Dynamics under $Q_\theta$

Under $Q_\theta$: 

$$dX_t = (AX_t + e_p\sigma_t \theta_t)dt + e_p\sigma_t dB^\theta_t$$  \hspace{1cm} (6)$$

with explicit dynamics, for $s \geq t \geq 0$:

$$X_s = \exp \{A(s - t)\} x + \int_t^s \exp \{A(s - u)\} e_p\sigma_u \theta_u du$$

$$+ \int_t^s \exp \{A(s - u)\} e_p\sigma_u dB^\theta_u$$ \hspace{1cm} (7)$$

Localizing Temperature Residual
CAT Futures

For $0 \leq t \leq \tau_1 < \tau_2$:

$$F_{\text{CAT}}(t, \tau_1, \tau_2) = \mathbb{E}^{Q_0} \left[ \int_{\tau_1}^{\tau_2} T_s ds | \mathcal{F}_t \right]$$

$$= \int_{\tau_1}^{\tau_2} \Lambda_u du + a_{t, \tau_1, \tau_2} X_t + \int_t^{\tau_1} \theta_u \sigma_u a_{t, \tau_1, \tau_2} e_p du$$

$$+ \int_{\tau_1}^{\tau_2} \theta_u \sigma_u e_1^\top A^{-1} \left[ \exp \left\{ A(\tau_2 - u) \right\} - I_p \right] e_p du \quad (8)$$

with $a_{t, \tau_1, \tau_2} = e_1^\top A^{-1} \left[ \exp \left\{ A(\tau_2 - t) \right\} - \exp \left\{ A(\tau_1 - t) \right\} \right]$, $I_p : p \times p$

identity matrix

Benth et al. (2007)

Localizing Temperature Residual
Localized Temperature Risk Analysis

Local Temperature Risk

Normality of $\varepsilon_t$ requires estimating the function $\theta(t) = \{\Lambda_t, \sigma_t^2\}$ with $t = 1 \cdots 365$ days, $j = 0 \cdots J$ years. Recall:

\[
X_{t,j} = T_{t,j} - \Lambda_t
\]

\[
X_{t,j} = \sum_{l=1}^{L} \beta_l X_{t-l,j} + \sigma_t \varepsilon_{t,j}
\]

\[
\varepsilon_{t,j} \sim N(0,1), \text{i.i.d.}
\]  

(9)

where $N(0,1)$ is the standard normal cdf.
Adaptation Scale

Fix $s \in 1, 2, \ldots, 365$, sequence of ordered weights:

$\mathcal{W}_s^{(k)} = (w(s, 1, h_k), w(s, 2, h_k), \ldots, w(s, 365, h_k))^\top$.

Define $w(s, t, h_k) = K_{h_k}(s - t), (h_1 < h_2 < \ldots < h_K)$.

Local likelihood:

$$
\hat{\epsilon}_{365j+t} = X_{365j+t} - \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l}
$$

$$
\tilde{\theta}^k(s) = \arg\min_{\theta \in \Theta} \sum_{t=1}^{365} \sum_{j=0}^{J} \left\{ \log(2\pi\theta)/2 + \hat{\epsilon}_{t,j}^2/2\theta \right\} w(s, t, h_k),
$$

$\overset{\text{def}}{=} \arg\min_{\theta \in \Theta} -L(\mathcal{W}_s^{(k)}, \theta)$
Localized Temperature Risk Analysis

\[ \tilde{\theta}^k(s) = \frac{\sum_{t,j} \hat{\varepsilon}^2_{t,j} w(s, t, h_k)}{\sum_{t,j} w(s, t, h_k)} \]

\[ = \frac{\sum_{t} \hat{\varepsilon}^2_t w(s, t, h_k)}{\sum_{t} w(s, t, h_k)} \]

with 
\[ \hat{\varepsilon}_t \overset{\text{def}}{=} (J + 1)^{-1} \sum_{j=0}^{J} \hat{\varepsilon}^2_{t,j}. \]

For Taipei, daily average temperature 19730101 – 20081230, \( J = 36 \).
Mirror Observations

To avoid the boundary problem, use mirrored observations:

Assume \( h_K < 365/2 \), then the observations look like
\[
\hat{\varepsilon}_2^{364}, \hat{\varepsilon}_2^{363}, \ldots, \hat{\varepsilon}_2^0, \hat{\varepsilon}_2^1, \ldots, \hat{\varepsilon}_2^{730},
\]
where
\[
\hat{\varepsilon}_t^2 \overset{\text{def}}{=} \hat{\varepsilon}_{365+t}^2, -364 \leq t \leq 0
\]
\[
\hat{\varepsilon}_t^2 \overset{\text{def}}{=} \hat{\varepsilon}_{t-365}^2, 366 \leq t \leq 730
\]
Parametric Exponential Bounds

\[ L(W^{(k)}, \theta, \theta^*) \overset{\text{def}}{=} L(W^{(k)}, \tilde{\theta}) - L(W^{(k)}, \theta^*) = NK(W^{(k)}, \tilde{\theta}, \theta^*) \]

\[ K(W^{(k)}, \theta, \theta^*) = -\{\log(\theta/\theta^*) + 1 - \theta^*/\theta\}/2 \]

where \( K\{\theta, \theta^*\} \) is the Kullback Leibler divergence between \( \theta \) and \( \theta^* \) and \( N = 365 \times J \). For any \( z > 0 \),

\[ P_{\theta^*}\{L(W^{(k)}, \tilde{\theta}, \theta^*) > z\} \leq 2 \exp(-z) \]

\[ E_{\theta^*} L(W^{(k)}, \tilde{\theta}, \theta^*)^r \leq r^r, \]

where \( r = 2r \int_{z \geq 0} z^{r-1} \exp(-z) dz \).
Localized Temperature Risk Analysis

LMS Procedure

Construct an estimate \( \hat{\theta} = \hat{\theta}(s) \), on the base of \( \tilde{\theta}_1(s), \tilde{\theta}_2(s), \ldots, \tilde{\theta}_K(s) \).

- Start with \( \hat{\theta}_1 = \tilde{\theta}_1 \).
- For \( k \geq 2 \), \( \tilde{\theta}_k \) is accepted and \( \hat{\theta}_k = \tilde{\theta}_k \) if \( \tilde{\theta}_{k-1} \) was accepted and
  \[
  L(W^{(k)}, \tilde{\theta}_\ell, \tilde{\theta}_k) \leq z_\ell, \ell = 1, \ldots, k - 1
  \]

\( \hat{\theta}_k \) is the the latest accepted estimate after the first \( k \) steps.
Localized Temperature Risk Analysis

Illustration

Localizing Temperature Residual
"Propagation" Condition

A bound for the risk associated with first kind error:

$$\mathbb{E}_{\theta^*} \left| L(W^{(k)}, \tilde{\theta}_k, \hat{\theta}_k) \right|^r_{r_r} \leq \alpha$$  \hspace{1cm} (10)

where \( k = 1, \ldots, K \) and \( r_r \) is the parametric risk bound.
Sequential Choice of Critical Values

Consider first only $\z_1$ letting $\z_2 = \ldots = \z_{K-1} = \infty$. Leads to the estimates $\hat{\theta}_k(\z_1)$ for $k = 2, \ldots, K$.

The value $\z_1$ is selected as the minimal one for which

$$\sup_{\theta^*} \frac{\|L\{W^{(k)}, \tilde{\theta}_k, \hat{\theta}_k(\z_1)\}\|_r}{r_r} \leq \frac{\alpha}{K-1}, \quad k = 2, \ldots, K.$$ 

Set $\z_{k+1} = \ldots = \z_{K-1} = \infty$ and fix $\z_k$ lead the set of parameters $\z_1, \ldots, \z_k, \infty, \ldots, \infty$ and the estimates $\hat{\theta}_m(\z_1, \ldots, \z_k)$ for $m = k + 1, \ldots, K$. Select $\z_k$ s.t.

$$\sup_{\theta^*} \frac{\|L\{W^{(k)}, \tilde{\theta}_m, \hat{\theta}_m(\z_1, \z_2, \ldots, \z_k)\}\|_r}{r_r} \leq \frac{k\alpha}{K-1},$$

$m = k + 1, \ldots, K$. 

Localizing Temperature Residual
Critical Values

Figure 9: Simulated CV with $\theta^* = 1$, $r = 0.5$, $MC = 5000$ with $\alpha = 0.3$, 0.5, 0.8 (left), with different bandwidth sequences (right).
Bound for Critical Values

Suppose $0 < \mu \leq h_{k-1}/h_k \leq \mu_0 < 1$.

Let $\theta(.) = \theta^*$, for all $t \in (0, 365)$. There is a constant $a_0 > 0$ depending on $r$ and $\mu_0, \mu$, s.t.

$$\delta_k = a_0 \log K + 2 \log(nh_k/\alpha) + 2r \log(h_K/h_k)$$

ensures (10).
"Small Modeling Bias" (SMB)

Condition:

\[
\Delta(W^{(k)}, \theta) = \sum_{t=1}^{365} \mathcal{K}\{\theta(t), \theta\} \mathbf{1}\{w(s, t, h_k) > 0\} \\
\leq \Delta, \forall k < k^* \tag{11}
\]

\(k^*\) is the maximum \(k\) satisfying the SMB condition.

Property:

For any estimate \(\tilde{\theta}_k\) and \(\theta\) satisfying SMB, it holds:

\[
E_{\theta(.)} \log\{1 + |L(\tilde{\theta}_k, \theta)|^{r/r_r}\} \leq \Delta + 1
\]
"Stability" Property

**Stability**: the attained quality of estimation during "propagation" can not get lost at further steps.

\[
L(W^{(k*)}, \tilde{\theta}_k^*, \hat{\theta}_{\hat{k}}) \mathbf{1}_{\{\hat{k} > k^\star\}} \leq \delta_{k^*}
\]
"Oracle" Property

Theorem

Let \( \Delta(W^{(k)}, \theta) \leq \Delta \) for some \( \theta \in \Theta \) and \( k \leq k^* \). Then

\[
E_{\theta(\cdot)} \log \left\{ 1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \theta)|_r}{r_r(W^{(k^*)})} \right\} \leq \Delta + 1
\]

\[
E_{\theta(\cdot)} \log \left\{ 1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \hat{\theta})|_r}{r_r(W^{(k^*)})} \right\} \leq \Delta + \alpha
\]

\[
+ \log\left\{ 1 + \frac{\delta^*_k}{r_r} \right\}
\]
Figure 10: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Beijing, $\alpha = 0.3$, $r = 0.5$

Localized Temperature Residual
Figure 11: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Kaoshiung, $\alpha = 0.3$, $r = 0.5$
Figure 12: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Berlin, $\alpha = 0.3$, $r = 0.5$
Figure 13: Q-Q Plot for squared residuals from Berlin, Kaoshiung, Beijing
## Normalized Residual

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<th>Bandwidth</th>
<th>KS</th>
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Table 4: P-value of normality tests for fixed bandwidth curve (FB), adaptive bandwidth curve (AB), adaptive smoothed bandwidth curve (ABS)
# Tokyo & Osaka AAT Future Prices

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<th>City</th>
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Table 5: Osaka AAT contracts listed at CME on 20090130. Source: Bloomberg. CME\(^1\) prices of AAT Futures as listed on CME, \(^2\) AAT index values computed from the realized temperature data and with adaptive bandwidth \(F_{AAT,\text{loc}}\)
Conclusions and further work

- Temperature risk stochastics closer to Wiener process when applying adaptive methods
- Better estimates of $\lambda_t$ and $\sigma_t$ lead to fair price $\rightarrow$ pure MPR
Localized Temperature Risk Analysis

References

F.E. Benth and J.S. Benth and S. Koekebakker
Putting a Price on Temperature

F.E. Benth and W.K. Härdle and B. López Cabrera
Pricing of Asian Temperature Risk

P.J. Brockwell
Continuous-time ARMA Processes
Localized Temperature Risk Analysis

References

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*Implied Market Price of Weather Risk*
Submitted to Journal of Applied Mathematical Finance, 2010

U. Horst and M. Müller
*On the Spanning Property of Risk Bonds Priced by Equilibrium*

W.K. Härdle, J. Franke, C.M. Hafner
*Statistics of Financial Markets: 3rd edition*
Springer Verlag, Heidelberg, 2011
Localizing Temperature Risk

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http://www.case.hu-berlin.de
Appendix A

Li-McLeod Portmanteau Test—modified Portmanteau test statistic $Q_L$ to check the uncorrelatedness of the residuals:

$$Q_L = n \sum_{k=1}^{L} r_k^2(\hat{\varepsilon}) + \frac{L(L+1)}{2n},$$

where $r_k$, $k = 1, \ldots, L$ are values of residuals ACF up to the first $L$ lags and $n$ is the sample size. Then,

$$Q_L \sim \chi^2_{(L-p-q)}$$

$Q_L$ is $\chi^2$-distributed on $(L - p - q)$ degrees of freedom where $p,q$ denote AR and MA order respectively and $L$ is a given value of considered lags.
WD pricing models

1. General Equilibrium Theory for incomplete markets: Indifference, Marginal, Market Clearing
2. Pricing via no arbitrage arguments: adequate equivalent martingale measure
$X_t$ can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$X_{1(t+2)} \approx (2 - \alpha_1)X_{1(t+1)} + (\alpha_1 - \alpha_2 - 1)X_{1t} + \sigma_t (B_{t-1} - B_t)$$

For $p = 3$,

$$X_{1(t+3)} \approx (3 - \alpha_1)X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3)X_{1(t+1)} + (-\alpha_1 + \alpha_2 - \alpha_3 + 1)X_{1t} + \sigma_t (B_{t-1} - B_t)$$

Localizing Temperature Residual
Proof $CAR(3) \approx AR(3)$

Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- assume a time step of length one $dt = 1$
- substitute iteratively into $X_1$ dynamics
Proof $\text{CAR}(3) \approx \text{AR}(3)$:

\[
\begin{align*}
X_1(t+1) - X_1(t) &= X_2(t)dt \\
X_2(t+1) - X_2(t) &= X_3(t)dt \\
X_3(t+1) - X_3(t) &= -\alpha_1 X_1(t)dt - \alpha_2 X_2(t)dt - \alpha_3 X_3(t)dt + \sigma_t \epsilon_t \\
X_1(t+2) - X_1(t+1) &= X_2(t+1)dt \\
X_2(t+2) - X_2(t+1) &= X_3(t+1)dt \\
X_3(t+2) - X_3(t+1) &= -\alpha_1 X_1(t+1)dt - \alpha_2 X_2(t+1)dt \\
&\quad -\alpha_3 X_3(t+1)dt + \sigma_{t+1} \epsilon_{t+1} \\
X_1(t+3) - X_1(t+2) &= X_2(t+2)dt \\
X_2(t+3) - X_2(t+2) &= X_3(t+2)dt \\
X_3(t+3) - X_3(t+2) &= -\alpha_1 X_1(t+2)dt - \alpha_2 X_2(t+2)dt \\
&\quad -\alpha_3 X_3(t+2)dt + \sigma_{t+2} \epsilon_{t+2}
\end{align*}
\]
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Table 6: Seasonality estimates \( \hat{\lambda}_t \) of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg.
**AR(p):**  
\[ X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t \]

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Table 7: Coefficients of AR(p), Model selection: AIC

The long memory diagnosis can be replicated by a short memory process with structural breaks!
Calibration of Seasonal Variance: $\sigma_t^2$

Table 8: First 7 Coefficients of $\sigma_t^2$ and GARCH($p = 1, q = 1$). The coefficients in black are significant at 1% level.
Figure 14: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature difference between adaptive and fixed bandwidth (lower), Beijing, $\alpha = 0.3$, $r = 0.5$

Localizing Temperature Residual
Figure 15: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature; difference between adaptive and fixed bandwidth (lower), Kaoshiung, $\alpha = 0.3$, $r = 0.5$
Figure 16: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature; difference between adaptive and fixed bandwidth (lower), Berlin, $\alpha = 0.3$, $r = 0.5$

Localizing Temperature Residual