Pricing and Hedging Strategies for Contingent Claims in an Incomplete Hybrid Emissions Market

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General Introduction

Context

- Kyoto Protocol (1997): Emissions in developed countries
  - Reduced
  - Stabilized

- Regulator: Control their national emissions
  - Corporate: Additional risk
  - Consumer: Erosion in purchasing power

- Most implemented instruments policies:
  - Emissions tax: Finland (1990), Sweden (1991), Quebec (2007),...
General Introduction

Cap-and-Trade Market Mechanism

- Ceiling for emissions
- Compliance period
- Market: price to comply with emission target
- Least cost: internal abatement or acquisition of allowances
- Trading between: Regulated emitters, Non-regulated emitters, Non-emitters
General Introduction

EU ETS Market

Figure 1: Futures prices for Dec 2009-14 from April 2008 to December 2009 (Source: Bloomberg)
General Introduction

EU ETS Market

- Auctioning up to 10% of total emissions in Phase II (Article 10 of the EU Directives)

- Point Carbon 2010 survey: 51% sold some surplus EUAs

- Short: power/heat sector

- Long: pulp/paper and cement/lime/glass sectors
General Introduction

Questions

- Is the cap-and-trade system the most cost effective policy instrument?
- Does it force emissions reductions?
- For which market design do price signals best describe the true cost of emitting a tonne of carbon?
- How can we protect regulated companies and consumers in the transition to a clean energy economy?
- Can we avoid carbon leakage?
Market Design

Hybrid system

• Removes the deficiencies of both the cap-and-trade and the carbon tax markets.

• Protects the economy by fixing a safety valve price.

• Special case: Cap-and-trade market.

Market Design

100% auctioning

- Eliminate windfall profits (Woerdman, Couwenberg and Nentjes, 2009).
- Incentives to innovate (Cramton and Kerr, 1998)
- Stable long-term price signal (Hepburn et al., 2006)
- Distribute % income to
  - final consumer as tax reduction
  - avoid carbon leakage
  - invest in green projects
## Market Design

**Principle market players**

<table>
<thead>
<tr>
<th>Regulator</th>
<th>Emitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Incomplete information: Abatement cost and emission quantity</td>
<td>- Abatement cost available</td>
</tr>
<tr>
<td>- Sets:</td>
<td>- Accurate emission prediction</td>
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<tr>
<td>- Auctioning price $P_0$</td>
<td>- Avoid paying $P_0$</td>
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<tr>
<td>- Initial endowment $N_0$</td>
<td>- Buy allowances $\leq N_0$</td>
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<tr>
<td>- Length of compliance periods</td>
<td></td>
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<tr>
<td>- Penalty $\pi$</td>
<td></td>
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</tbody>
</table>
Two period Market Model

- Compliance dates $T_1$ and $T_2$, $T_1 < T_2$
- Banking allowed: Do not affect market position at $T_1$
- Borrowing Forbidden
- Insufficient certificates at $T_1$: Later delivery + Penalty $\pi$ to pay at $T_1$
- Safety valve price $P_{max}$
- Adjust market parameters at time $T_1^+$

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Two period Market Model

Define

- The discounted price process of the future contracts:
  - $X^1_t$ matures at $T_1$
  - $X^2_t$ matures at $T_2$

- No-arbitrage condition: $X^1_t \leq \pi e^{-r(T_1-t)} + X^2_t$

- Stopping time $\tau$
  \[
  \tau(\omega) = \min\{t/X^1_t(\omega) = P_{max}\}
  \]

- $T_1$-contingent claim: $H = f(X) \in L^2(P)$
Two period Market Model

- \( \forall t \geq \tau(\omega), X^1_t = P_{max} \)

- \( \tau(\omega) < T_1 \)
  - \( X^2_{T_1} = P_{max} - \pi \)
  - \( H(\omega) = f(P_{max}) \)

- \( \tau(\omega) \geq T_1 \)
  - Short: \( X^2_{T_1} < X^1_{T_1} \)
    \[ \Rightarrow H(\omega) = f(\min(X^1_{T_1}(\omega), \pi + X^2_{T_1}(\omega), P_{max})) \]
  - Not short: \( X^2_{T_1} \geq X^1_{T_1} \)
    \[ \Rightarrow H(\omega) = f(X^1_{T_1}(\omega)) \]
Two period Market Model

Effective payoff

\[ H = \mathbb{I}_{\tau \geq T_1} \]

\[ \quad \left[ \mathbb{I}_{X_{T_1}^2 < X_{T_1}^1} f(\min(X_{T_1}^1, \pi + X_{T_1}^2, P_{max})) \right] \]

\[ \quad + \mathbb{I}_{X_{T_1}^2 \geq X_{T_1}^1} f(X_{T_1}^1) \]

\[ \quad + \mathbb{I}_{\tau < T_1} f(P_{max}) \]

Example: \( H := T_1 \)-Call option at strike \( K \) written on \( X_t^1 \)

- \( f(X) = (X - K)^+ \)
- Depends on \( X_{T_1}^2 \)
Pricing and Hedging Solution

Mean reversion Jump diffusion dynamic (Daskalakis et al., 2009)

- \( dX^i_t = \theta_i dt + X^i_t (\mu_i dt + \sigma_1 dw_{1t} + \sigma_2 dw_{2t} + \varphi_{i1} dN_{1t} + \varphi_{i2} dN_{2t}) \), \( X^i_0 > 0, \ \varphi_{i1} > -1 \)
- \( (N_{1t}, N_{2t})' \): Poisson process with intensity \( \nu = (\nu_1, \nu_2)' \)
- \( (w_{1t}, w_{2t})' \): independent Brownian motions

Probability space \((\Omega, \mathcal{F}, P)\)

- Complete
- \( \mathcal{F}_0 \) is trivial and contains all null sets of \( \mathcal{F} \)
- \( \mathcal{F}_t \): \( P \)-augmented right continuous filtration generated by \( w_t \) and \( N_t, \forall t \leq T_1 \)
Pricing and Hedging Solution

Market incomplete

- Existence of intrinsic risk
- Equivalent martingale measure $\mathcal{M}^e(X) \neq \{\}$

Doob-Meyer Decomposition

- $X_t^i = X_0^{i*} + M_t^i + A_t^i, \ i = 1..2$
- $M_t^i$: Local $P$-martingale
- $A_t^i$: predictable process with finite variation
- $X_0^{i*}$: $\mathcal{F}_0$-measurable
Pricing and Hedging Solution

Define

- Space of square integrable martingales: $\mathcal{M}^2(P)$
- Dynamic strategies: $\phi = (\xi_t, \eta_t)_{0 \leq t \leq T}$
- Portfolio value process: $V_t = \xi' \cdot X_t + \eta_t$
- Cumulative gains: $G_t(\xi) = \int_0^t \xi_s dX_s$
- Cumulative cost: $C_t = V_t - G_t(\xi)$
- Risk: $R_t(\phi) = E((C_T(\phi) - C_t(\phi))^2)$
- Optimality Equation: $\phi^*$ s.t. $R_t(\phi^*) \leq R_t(\phi)$ for all admissible $\phi$
Pricing and Hedging Solution

Föllmer-Schweizer Decomposition

\[ C(\phi) \in \mathcal{M}^2(P), \quad C(\phi) \perp M^i \text{ under } P \]

\[ \uparrow \]

\[ H = H_0 + \int_0^T \xi^H \cdot dX_t + L^H_T, \quad P \text{ a.s} \]

where

- \( H_0 \in \mathbb{R} \),
- \( \xi^H \in \Theta \),
- \( L^H \in \mathcal{M}^2(P) \) and \( L^H \perp M^i \),
- \( \Theta = \{ (\xi)_t/\mathbb{R}^2 \text{ - predictable process, (} E[\int_0^T \xi'_t d < M>_t \xi_t)]^{1/2} < \infty, \quad \text{and } E[(\int_0^T \xi'_t dA_t)^2] < \infty \} \)
Pricing and Hedging Solution

Let

\[
\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix},
\]

\[
\Lambda = \begin{pmatrix} \sigma_{11}^2 + \sigma_{12}^2 & \sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22} \\ \sigma_{11}\sigma_{21} + \sigma_{22}\sigma_{12} & \sigma_{21}^2 + \sigma_{22}^2 \end{pmatrix},
\]

\[
\Xi = \begin{pmatrix} \varphi_{11}^2\nu_1 + \varphi_{12}^2\nu_2 & \varphi_{11}\varphi_{21}\nu_1 + \varphi_{12}\varphi_{22}\nu_2 \\ \varphi_{21}\varphi_{11}\nu_1 + \varphi_{22}\varphi_{12}\nu_2 & \varphi_{21}^2\nu_1 + \varphi_{22}^2\nu_2 \end{pmatrix},
\]

\[
\alpha = (\Lambda + \Xi)^{-1}(\mu + \Phi\nu).
\]
Pricing and Hedging Solution

Mean-variance tradeoff process

Define

\[ \hat{\lambda}_t^i := \frac{1}{X_t^i} \alpha_i, \quad \text{for } i = 1, 2 \]

\[ \hat{K}_t = \int_0^t \hat{\lambda}'_s d < M >_s \hat{\lambda}_s \]

\( \hat{K}_t \) Properties

- Deterministic
- Uniformly bounded in \((t, \omega)\)

\[ \Rightarrow \exists! \text{ F-S Decomposition (Monat and Stricker, 1995)} \]
Pricing and Hedging Solution

Minimal Martingale Measure $\hat{P}$

Density process

$$Z_t = \varepsilon(-\int_0^t \hat{\lambda}_s dM_s)_t, \quad 0 \leq t \leq T_1,$$

where

$$\varepsilon(X) = 1 + \int_0^t \varepsilon(X)_s - dY_s, \quad 0 \leq t \leq T_1.$$

- $Z_t > 0$ if $\exists \delta/(\alpha \Phi)_i < 1 - \delta$ (Arai, 2004)
- $Z_t \in \mathcal{M}^2(P)$ (Choulli et al., 1998)
Pricing and Hedging Solution

Under $\hat{P}$,

$$V_t = \hat{E}[H|\mathcal{F}_t]$$

Pricing Procedure

- Dynamics under $\hat{P}$

$$dX^i_t = \theta_i dt + X^i_t((\mu_i - \sigma_{i1}(\alpha_1\sigma_{11} + \alpha_2\sigma_{21}) - \sigma_{i2}(\sigma_{12}\alpha_1 + \sigma_{22}\alpha_2))dt$$

$$+ \sigma_{i1}dw^P_{1t} + \sigma_{i2}dw^P_{2t} + \varphi_{i1}dN^P_{1t} + \varphi_{i2}dN^P_{2t}), \quad X^i_0 > 0$$

- $(w^P_{1t}, w^P_{2t})'$: $\hat{P}$-standard Brownian motions

- $N^P_t = (N^P_{1t}, N^P_{2t})'$: Poisson process under $\hat{P}$ with intensity

$$\nu^P = (\nu_1(1 - \alpha_1\varphi_{11})(1 - \alpha_2\varphi_{21}), \nu_2(1 - \alpha_1\varphi_{12})(1 - \alpha_2\varphi_{22}))'$$

- $V_0 = \hat{E}[H]$
Pricing and Hedging Solution

Pricing $T_2$-contingent claim

- Market efficiency vs Structural adjustment at $T_1^+$

- Pricing under Larger filtration $\tilde{\mathcal{F}} \supseteq \mathcal{F}$ such that $\forall t \leq T_1, \tilde{\mathcal{F}}_t = \mathcal{F}_t$

- Two period setting: $H = f(X_{T_2}^2)$

- Given $H$ is attainable under $\tilde{\mathcal{F}}$:

$$H = H_0 + \int_0^{T_2} \tilde{\xi}_s dX_s^2$$

where

$\tilde{\xi}$: $\tilde{\mathcal{F}}$-measurable admissible strategy
$H_0$: $\tilde{\mathcal{F}}_0$-measurable
Pricing and Hedging Solution

Pricing $T_2$-contingent claim

- For $t < T_1$: $(\mathcal{F})_{t \geq 0}$ available $\implies$ intrinsic risk for $H$

\[
H = H_0 + \int_0^{T_1} \tilde{\xi}_s dX_s^2 + \int_{T_1^+}^{T_2} \tilde{\xi}_s dX_s^2
\]

Full hedge \hspace{2cm} Partial hedge

\[
\int_{T_1^+}^{T_2} \tilde{\xi}_s dX_s^2 = \int_{T_1^+}^{T_2} \xi_s dX_s^2 + N_{T_2}
\]

where $\xi_s : \mathcal{F}_s$-measurable, $N_{T_2} \in \mathcal{L}^2(\Omega, \mathcal{F}_{T_2}, \mathbb{P})$ and $N_t \perp M_t^2$
Pricing and Hedging Solution

Pricing $T_2$-contingent claim

- For $t < T_1$: $(\mathcal{F})_{t \geq 0}$ available $\implies$ intrinsic risk for $H$

\[
H = H_0 + \int_0^{T_1} \tilde{\xi}_s dX^2_s + \int_{T_1^+}^{T_2} \tilde{\xi}_s dX^2_s
\]

- Full hedge
- Partial hedge

Intrinsic risk

\[
\int_{T_1^+}^{T_2} \tilde{\xi}_s dX^2_s = \int_{T_1^+}^{T_2} \xi_s dX^2_s + N_{T_2}
\]

where $\xi_s : \mathcal{F}_s$-measurable, $N_{T_2} \in \mathcal{L}^2(\Omega, F_{T_2}, P)$ and $N_t \perp M_t^2$
Concluding remarks

- Quadratic Hedging
  
  \[
  \min E[(H - V_0 - \int_0^T \xi_s dX_s)^2]
  \]

- Get around incomplete information by Super-Hedging
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