Optimal Switching Games in Emissions Trading

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Outline

1. Cap-and-Trade: Producer Perspective
2. Switching Games
3. Correlated Equilibria in $CO_2$ markets
4. Numerical Illustrations
5. Open Problems
Emissions Trading

- Major new initiatives are underway to introduce $CO_2$ cap-and-trade schemes that will create new commodity markets.
- **AB32** proposal in California; various federal proposals; EU ETS.
- The estimated size of the market is in the hundreds of billions or even trillions of dollars.
- Key regulatory details are still unresolved and undergo active public debate.
- Crucial to understand the financial implications of these initiatives on energy producers.
A New Commodity Market

Compared to existing markets, cap-and-trade is fundamentally different:

- A finite resource is initially allocated and subject to exhaustion.
- A well-defined horizon (e.g. 1 year) exists for each allocation.
- The permit prices converge to deterministic values as horizon approaches.
- Price formation is driven by participant strategies: must be endogenous to any model.
- Game-theoretic aspects emerge in the emissions market.
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Effect on Producers

- The foremost constituency affected by cap-and-trade would be energy producers.
- The net profit of energy production would change from the spark-spread to the clean spread.
- Commodity prices (input fuel, output fuel) are stochastic.
- Must take into account (dynamic) strategies of other participants.
- Feedback between production policies and carbon prices.
Related Literature

- **Real Options**: Dixit and Pindyck (1994), Dockner et al. (2000).
- **Analysis of Cap-and-Trade**: Carmona et al. (2008, 2009), Cetin and Verschuere (2008), ...
- **Stochastic Differential Games**: Bensoussan, Friedman, Hamadène, Lepeltier,...
Model Setup

- We focus on the **timing optionality** within a real-options framework.
- Consider a **duopoly** – two producers (representing different sets of power plants).
- Each one sells electricity into a stochastic market at price $P_t$.
- Need emission permits to produce. Must buy $CO_2$ permits on the market at price $X_t$.
- Take a reduced-form **price-impact** model for $(X_t)$ (do not explicitly model the remaining supply of permits).
- Simplify the strategy set: at each time epoch either produce, or stay offline, $\xi_i(t) \in \{0, 1\}$.
- Each producer’s policy influences **changes** in $X$; $\Rightarrow$ the scheduling decisions of agents affect each other.
- Discrete-time model.
Objective

- \((P_t)\) is exogenously given.
- Mean increments of \((X_t)\) are controlled by \(\vec{\xi}(t)\); correlated with increments of \((P_t)\).
- Changes in \(\xi_i\) are expensive: fixed switching costs \(K_i,\xi_i(t-1),\xi_i(t)\); induce inertia and hysteresis.
- Fixed horizon \(T\): expiration date of the permits.
- Each producer attempts to maximize

\[
V^i(0, p, x, \vec{\zeta}) = \mathbb{E} \left[ \sum_{t=1}^{T} \left( \xi_i(t)(a_i P_t - b_i X_t^{(\xi)} - c_i) - K_i,\xi_i(t-1),\xi_i(t) \right) \right].
\]
Dynamic Decision-Making

- At each step, each producer decides whether to produce or not.
- The chosen action results in immediate date $t$-payoff, as well as different continuation values on $[t + 1, T]$.
- Leads to a repeated $2 \times 2$ stochastic game.
- Bellman’s Principle is replaced by a game Nash Equilibrium (NE).
- Pure Nash equilibria might not exist.
- Existence: Need mixed equilibria.
- Might also have multiple Nash equilibria.
- Uniqueness: equilibrium refinement.
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Classification of 1-Period $2 \times 2$ Games

Three equivalence classes:

- A single dominating pure equilibrium (unanimity).
- A competitive game (essentially zero-sum) which admits a unique mixed Nash eqm.
- A (anti-) coordination game which admits two pure eqm’s, a mixed one and a continuum of correlated eqm’s – “battle-of-the-sexes” as above.
- Profitable for each one to emit separately; not profitable to emit together.
- Which producer will yield??
Correlated Equilibria

- Nash equilibrium: given the eqm strategy of the other player, maximizes your expected payoff.
- Overall payoff distribution is a product measure on the payoff space.
- A correlated equilibrium \((\gamma_{jk})\) is a general probability distribution on the payoff space. Known to all and fixed in advance.
- Achieved by introducing a third (fictitious) agent, (regulator).
- The regulator sends a private signal \(\mu_i(\gamma) \in \{0, 1\}\) to player \(i\).
- Given the signal (and implied strategy of the second player), optimal to act according to \(\mu_i\).
- Conditional on signal, equilibrium action is pure.
Stopping Games

- A stopping game: each agent chooses a stopping time $\tau_i$, $i = 1, 2$. Stopping corresponds to action '1'.
- Payoff structure ($\mathcal{Z}$); agent $i$ receives $(\tau \equiv \tau_1 \land \tau_2)$

\[
\tau - 1 \sum_{t=0}^{\tau-1} \mathcal{Z}_i^{00}(t)) + \mathcal{Z}_i^{10}(\tau)1_{\{\tau_i<\tau_j\}} + \mathcal{Z}_i^{01}(\tau)1_{\{\tau_i>\tau_j\}} + \mathcal{Z}_i^{11}(\tau)1_{\{\tau_i=\tau_j\}}.
\]

- Starting with known values at $T$ move back in time; each period yields a 2-by-2 game with payoffs corresponding to conditional expectation of next-period value.

- Let $Val_\gamma(\mathcal{Z}_t)$ be an equilibrium of a 2-by-2 one-period game with payoffs

\[
\mathcal{Z}_t = \begin{pmatrix}
(\tilde{\mathcal{Z}}_1(t), \tilde{\mathcal{Z}}_2(t)) & (\mathcal{Z}_1^{01}(t), \mathcal{Z}_2^{01}(t)) \\
(\mathcal{Z}_1^{10}(t), \mathcal{Z}_2^{10}(t)) & (\mathcal{Z}_1^{11}(t), \mathcal{Z}_2^{11}(t))
\end{pmatrix}.
\]

- Stopping game values solve $(V_1(t), V_2(t)) = Val_\gamma(\mathcal{Z}_t)$, with

\[
(\tilde{\mathcal{Z}}_1(t), \tilde{\mathcal{Z}}_2(t)) \equiv (\mathbb{E}[V_1(t + 1)|\mathcal{F}_t] + \mathcal{Z}_1^{00}(t), \mathbb{E}[V_2(t + 1)|\mathcal{F}_t] + \mathcal{Z}_2^{00}(t)).
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Nash Equilibria in Stopping Games

- To show existence of a pure Nash equilibrium need restrictive assumptions (e.g. Dynkin zero-sum games, non-zero-sum monotone games where $Z_{i}^{01} \leq Z_{i}^{11} \leq Z_{i}^{10}$).
- In general, must allow randomized stopping times.
- This is an $(\mathcal{F}_t)$-adapted stochastic process $p = (p_t)$ with $0 \leq p_t \leq 1$ a.s.
- $\tau(p) \triangleq \inf \{ t : \eta_t \leq p_t \}$ where $\eta_t \sim \text{Unif}(0, 1)$ i.i.d.. $p_t$ is the probability of stopping at date $t$, conditional on not stopping so far.
- $\tau(p)$ is not $(\mathcal{F}_t)$-adapted. Enlarge the filtration: $\tau(p)$ is a $(\mathcal{F}_t \vee \sigma(\eta_t))$-stopping time.
- Shmaya & Solan (2004): any discrete-time stopping game admits a mixed NE.
CE in Stopping Games

- **A correlation law** \((\gamma_{jk}(t))\) is a function of \((t, P_t, X_t)\) and fixed/known in advance. Gives a CE for any payoff structure \(Z_t\).
- At each state \(t\), agent \(i\) receives a private signal \(\mu_i(t; \gamma)\).
- Resulting randomized stopping time is \(\tau_i(\gamma)\). \(\tau_i, \tau_j\) are dependent!
- At each stage \(\gamma_{jk}(t)\) induces a CE — no incentive to deviate given \(\mu_i(t; \gamma)\).
- Admissible overall strategies are \(\mathcal{G}^i\)-stopping times \(\tau_i\), with \(\mathcal{G}^i_t = \sigma(P_s, X_s, \mu_i(s), 0 \leq s \leq t)\).
- Game is non-cooperative; no possibility of threats, etc. Even if deviate continue to receive future messages and no changes are made.
Switching Game I

- **We have a switching game.** This is a sequential stopping game: can repeatedly “stop” to alter production regimes in response to changing electricity prices, permit prices or other agent’s actions.

- **Player \(i\):** value function \(V_i(t, P_t, X_t, \tilde{\xi}_t)\).

\[
(V_1(t, \tilde{\zeta}), V_2(t, \tilde{\zeta})) = \text{Val}_{\gamma} \left( \frac{(V_1(t + 1, \tilde{\zeta}) - K_1, \zeta_1, V_2(t + 1, \tilde{\zeta}) - K_2, \zeta_2)}{V_1(t, \zeta_1, \zeta_2) - K_1, \zeta_1, V_2(t, \zeta_1, \zeta_2) - K_2, \zeta_2} \right)
\]

where \(Z_i(t) \triangleq (a_i P_t - b_i X_t - c_i) \zeta_i\).

- **Overall play:**
  - Observe current state \((P_t, X_t, \tilde{\xi}_t - 1)\);
  - Regulator carries out randomization;
  - Receive private signals \(\mu_i(t; \gamma)\);
  - Choose private actions;
  - Joint action \(\tilde{\xi}_t\) is revealed, update state variables for next period;
Switching Game II

- **Sketch of proof**: Restrict strategy sets so that agents can only use up to $(n, m)$ switches.

- Translates into an iterative stopping game with payoffs corresponding to $(n - 1, m)$, $(n, m - 1)$ or $(n - 1, m - 1)$ cases.

- Fixing the strategy of one player; the other player solves a switching problem with respect to the enlarged filtration $\mathcal{G}^i$.

- By definition of $\gamma$ this gives a CE in the switching game.

- Take $n, m \to \infty$ to obtain a coupled pair of value functions as above.
Digression: Single Player Case

- Fix the strategy of one producer and consider the optimization of the other one.
- This becomes an optimal switching problem as studied in Carmona-M.L. (2008).
- The price impact leads to significant hysteresis effect.
- If the price impact is severe enough, will always stay offline (or at least with very high probability) – “blockading”.
- From player’s 1 perspective, the actions of player 2 are randomized: continuation values are unknown, optimal stopping in “random environment”.
- Otherwise, standard optimal stopping problem in the enlarged filtration \( \mathcal{G}_t^i \) that incorporates CE.
Emissions Trading

Switching Games

Numerics

Conclusion
Numerical Solution

- To solve for the game values numerically need to
  - Be able to compute equilibria in $2 \times 2$ games;
  - Compute conditional expectations.

- Have explicit formulas for CE of $2 \times 2$ games (answer depends on CE choice).

- Need approximation; recall that $(P, X)$ have continuous space.

- Need to work with four different prob. measures $\mathbb{P}^\zeta$ due to the price impact.
Least Squares Monte Carlo

- Could use Markov Chain approximation, see Kushner (2007).
- To compute the conditional expectations, another robust algorithm is to use Monte Carlo simulation.
- Simulate paths of \((P, X)\) for each of the four possible emission regimes \(\zeta\).
- **Continuation values** are approximated through a cross-sectional regression.
- If the optimal decision is to switch to another regime, then use the approximate continuation value; else recursively update the future path-value.
- Extends the Longstaff-Schwartz method for American option pricing (a single optimal stopping problem).
- Straightforward extension to randomized stopping ... and to 2-player game.
Example

- \[ P_{t+1} = P_t \cdot \exp(2(50 - \log P_t) + 0.4\epsilon_t^P); \]
- \[ X_{t+1} = X_t \cdot \exp(3(\log(12 + 8\xi_1(t)) + 4\xi_2(t) - \log X_t) + 0.25\epsilon_t^X) \text{ with } \mathbb{E}[\epsilon_t^P \epsilon_t^X] = 0.6; \]
- Revenues: \[ Z_1(t) = P_t - 2X_t - 10; \]
  \[ Z_2(t) = 2P_t - X_t - 80; \]
- \( T = 1, 26 \text{ periods (}\Delta t = 1/26); \ K \equiv 0.2. \)
- Using the simulation solver:

<table>
<thead>
<tr>
<th>Correlation Law</th>
<th>( V_1(0, P_0, X_0) )</th>
<th>( V_2(0, P_0, X_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilitarian</td>
<td>5.30</td>
<td>4.14</td>
</tr>
<tr>
<td>Egalitarian</td>
<td>5.33</td>
<td>4.20</td>
</tr>
<tr>
<td>Preferential 1</td>
<td>5.39</td>
<td>4.11</td>
</tr>
<tr>
<td>Preferential 2</td>
<td>5.02</td>
<td>4.24</td>
</tr>
</tbody>
</table>
Date-$t$ Equilibria

**Figure:** Optimal game strategy $\xi^*$ as a function of $(P_t, X_t)$ for $t = 0.25$. Here $\zeta = (0, 0)$. The **green region** denotes the anti-coordination CE and the **red region** denotes the competitive mixed NE.
A Realized Equilibrium Path

Figure: Sample path of the controlled $X_t$, including the corresponding strategy $\xi^* \in \{00, 01, 10, 11\}$. The top left panel shows the cumulative P&L of each player; the bottom left panel shows the raw P&L for each time period. Finally, the right panel shows the evolution of the controlled $X_t$, as well as the implemented strategy $(\xi^1_t, \xi^2_t)$. Note as $\xi_t$ increases, emissions rise and $X_t$ tends to increase.
Conclusion

- Stochastic games naturally occur in studying oligopolies.
- The emission market would be a new important class of such problems.
- Investigate the simplest possible scenario where the game is non-trivial: a new model of an optimal switching game.
- Already the problems of equilibrium-refinement and computational tractability arise.
- **To Do:** incorporate initial permit allocations/trading of permits. Allow for endogenous price formation.
- Continuous time formulation of correlated equilibria in stopping games??
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M. Ludkovski
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Formal $2 \times 2$ Game

- Payoffs $H = \begin{pmatrix} (\alpha^{00}, \beta^{00}) & (\alpha^{01}, \beta^{01}) \\ (\alpha^{10}, \beta^{10}) & (\alpha^{11}, \beta^{11}) \end{pmatrix}$.

- A policy is $(\bar{\pi}, \bar{\rho})$ whence $\pi_i$ (resp. $\rho_j$) is the prob. that player 1 (player 2) chooses action $i$.

- Value of a policy to players is $Val(H; \bar{\pi}, \bar{\rho}) := \left( \sum_{i,j} \pi_i \rho_j \alpha^{ij} \right) \div \left( \sum_{i,j} \pi_i \rho_j \beta^{ij} \right)$.

- $\gamma = (\gamma^{ij})$ is a CE if

  $$\begin{cases} 
  \gamma^{00} \alpha^{00} + \gamma^{01} \alpha^{01} \geq \gamma^{00} \alpha^{10} + \gamma^{01} \alpha^{11}, \\
  \gamma^{00} \beta^{00} + \gamma^{10} \beta^{10} \geq \gamma^{00} \beta^{01} + \gamma^{10} \beta^{11}, \\
  \gamma^{11} \alpha^{11} + \gamma^{10} \alpha^{10} \geq \gamma^{11} \alpha^{01} + \gamma^{10} \alpha^{00}, \\
  \gamma^{11} \beta^{11} + \gamma^{01} \beta^{01} \geq \gamma^{11} \beta^{10} + \gamma^{01} \beta^{00}. 
  \end{cases}$$

- Leads to game values $Val_{\gamma}(H) := \left( \sum_{i,j} \gamma^{ij} \alpha^{ij} \right) \div \left( \sum_{i,j} \gamma^{ij} \beta^{ij} \right)$. 