Asset Financing with Credit Risk

Steven Golbeck and Vadim Linetsky

Northwestern University, Department of Industrial Engineering & Management Sciences

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This talk focuses on financing depreciating equipment, with particular focus on aircraft financing (aircraft mortgages) and their securitization (EETCs)
Airline Monitors forecast of total global aircraft deliveries in 2009-2030: US $4.728 trillion
Aircraft Financing Market

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- Export credit agencies (credit guarantees to private market): In the event of airline default, the guarantor (ECA) pays off the outstanding principal plus accrued interest and takes over the underlying asset backed loan. Similar to a CDS written on an asset backed loan.
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- Aircraft manufacturers through their credit operations
Risks in Aircraft Financing (and Leasing)

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- Model for the underlying asset: depreciating equipment with significant and time (age) dependent secondary market volatility
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Modeling the Risks
- Model for the underlying asset: depreciating equipment with significant and time (age) dependent secondary market volatility
- Model to evaluate secured asset financing (loans, leases, securitization vehicles, credit guarantees) consistently with unsecured financing (senior unsecured corporate debt, Credit Default Swaps (CDS))
What does the underlying asset look like?

- Ascend historical current market value (CMV) data for 14 models:
  
  B737-300 1984-99  Dash8-100 1983-2003
  B737-800 1997-2007  DC10-30 1972-88
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- From 1970 to mid-2008, 230 model/vintage time series of annual CMV appraisals, 3,528 data points
- Data normalized to 100 for initial appraisal and deflated by CPI to adjust for inflation
- For estimation, each time series (i.e. B737-300, vintage 1994 and DC10-30, vintage 1985) was considered to be a sample path of the same random variable (the asset value), and appraisals were binned by age (i.e. all appraisals for 3 year-old models were binned together)
Ascend CMV Histories (Inflation Adjusted)
Residual Value Curve (RVC): Mean Value as a Function of Age

- This is the Forward Curve under the $\mathbb{P}$-measure:

\[ F^\mathbb{P}(0, t) = \mathbb{E}^\mathbb{P}[S_t] \]

- Tracks the expected depreciation of the aircraft with age
We will later identify this with the asset volatility $\Sigma(t)$.
Stochastic Model for Depreciating Equipment

- Equipment is new at $t = 0$ with price $S_0$. Secondary Market used equipment price dynamics under the physical probability measure $\mathbb{P}$:

\[
dS_t = -D_t S_t dt + S_t \sigma_S(t) \cdot dW_t^\mathbb{P}
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$$dS_t = -D_t S_t dt + S_t \sigma_S(t) \cdot dW_t^\mathbb{P}$$

- Assuming the absence of arbitrage, under a change of measure $dW_t^\mathbb{P} = dW_t^\mathbb{Q} - \eta_t dt$, we obtain the risk-neutral ($\mathbb{Q}$) dynamics:

$$dS_t = (r_t - \ell_t) S_t dt + S_t \sigma_S(t) \cdot dW_t^\mathbb{Q}$$

$$\sigma_S(t) \cdot \eta_t = \ell_t - r_t - D_t$$

where $\ell_t$ is the stochastic default-free instantaneous lease rate (short lease rate).
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- If we assume that the equipment has a finite lifetime \( T \), at which point it is sold for its scrap value \( S_T \) (which we assume to independent of the model dynamics), we obtain that:

\[
S_t = S_T \mathbb{E}_t^\mathbb{Q} \left[ e^{\int_t^T (r_s - \ell_s) ds} \right]
\]
We assume that the stochastic short lease-rate follows a Vasicek process with time-dependent parameters under $\mathbb{Q}$:

$$d\ell_t = \kappa \ell(\theta(t) - \ell_t)dt + \sigma(t) \cdot dW^\mathbb{Q}_t,$$

where $\ell_t$ represents the lease rate, $\kappa$ and $\sigma(t)$ are time-dependent parameters, and $W^\mathbb{Q}$ is a Brownian motion under the risk-neutral measure $\mathbb{Q}$. The asset dynamics are therefore lognormal.
Stochastic Model for Depreciating Equipment

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\[
d\ell_t = \kappa_\ell (\theta_\ell(t) - \ell_t)dt + \sigma_\ell(t) \cdot dW^\mathbb{Q}_t,
\]

- Express \( \ell_t \) in terms of the OU process with zero long-run mean \( X^\ell_t \) and deterministic \( \Delta^\ell_t \):

\[
\ell_t = \Delta^\ell_t + X^\ell_t, \quad \Delta^\ell_0 = \ell_0, \quad X^\ell_0 = 0
\]

\[
dX^\ell_t = -\kappa_\ell X^\ell_t dt + \sigma_\ell(t) \cdot dW^\ell_t.
\]

\[
X^\ell_t = X^\ell_s + \int_s^t e^{-\kappa_\ell(t-u)} \sigma_\ell(u) \cdot dW^\ell_u,
\]
Stochastic Model for Depreciating Equipment

- We assume that the stochastic short lease-rate follows a Vasicek process with time-dependent parameters under $Q$:

\[ d\ell_t = \kappa_\ell (\theta_\ell(t) - \ell_t)dt + \sigma_\ell(t) \cdot dW^Q_t, \]

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- The asset dynamics are therefore lognormal and in terms of $X_\ell^t$ and the risk-neutral forward curve $F^Q(0,t)$:

\[ S_t = F^Q(0,t) e^{\frac{X_\ell^t}{\kappa_\ell} (1 - e^{-\kappa_\ell(T-u)}) - \frac{1}{2} \Sigma^2(t)} \]
Stochastic Model for Depreciating Equipment

This lognormal specification fixes the asset volatility function such that:

\[
\sigma_S(t) = \sigma_\ell(t, T) \equiv \frac{\sigma_\ell(t)}{\kappa_\ell} (1 - e^{-\kappa_\ell (T-t)}) \\
\Sigma^2(t) = \frac{1}{\kappa^2_\ell} (1 - e^{-\kappa_\ell (T-t)})^2 \int_0^t \sigma^2_\ell(s) e^{-2\kappa_\ell (t-s)} ds \\
S_t = F^Q(0, t) e^{\Sigma^2 z^Q - \frac{1}{2} \Sigma^2} \\
z^Q \sim \mathcal{N}(0, 1)
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\]

We estimate the model from the time series data on used aircraft prices, \(F^P(0, t) = \mathbb{E}^P[St]\), by exploiting the change-of-measure relationship:

\[
F^Q(0, t) = F^P(0, t)e^{-\eta \int_{0}^{t} \sigma_{\ell}(s, T)e^{-\kappa_{\ell}(t-s)} ds}
\]

which along with the estimated squared volatility function, yields that:

\[
S_t = F^P(0, t)e^{-\eta \int_{0}^{t} \sigma_{\ell}(s, T)e^{-\kappa_{\ell}(t-s)} ds} \frac{X_{\ell}}{e^{\kappa_{\ell}}(1-e^{-\kappa_{\ell}(T-u)}) - \frac{1}{2} \Sigma^2(t)}
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Intuition and Terminology

- From empirical data ($\mathbb{P}$-measure), we estimated $F^\mathbb{P}(0, t)$ expected future equipment value under $\mathbb{P}$. In the industry it is called the Residual Value Curve (RVC).
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- The RVC is the key input data used in the equipment leasing and financing industry. It estimates the expected economic depreciation of the equipment over its useful economic life. However, there is significant market price volatility in the secondary market for used equipment.

We observe the relationship between the forward curve $F^\mathbb{Q}(0, t)$ (expected future value under $\mathbb{Q}$) and the RVC:

$$F^\mathbb{Q}(0, t) = F^\mathbb{P}(0, t) e^{-\eta \int_0^t \sigma^\ell(s, T) e^{-\kappa^\ell(t-s)} ds}$$

We call this **Risk Adjusted Residual Value Curve (RA-RVC)**. We will imply the market price of risk $\eta$ from the market prices of Enhanced Equipment Trust Certificates (EETCs) - securitized aircraft mortgages which are publicly traded, and therefore provide an opportunity to estimate the MPR.
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$$R(\tau) = \min\{P(\tau) + I(\tau), S_{\tau} - TC\} + \pi \max\{P(\tau) + I(\tau) + TC - S_{\tau}, 0\}$$

$$= P(\tau) + I(\tau) - (1 - \pi) \max\{P(\tau) + I(\tau) + TC - S_{\tau}, 0\},$$

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We may therefore write the value of a loan of maturity $T^*$ as,

$$M = \mathbb{E} \left[ \sum_{k=0}^{N} e^{-\int_{0}^{t_k} r_s \, ds} C_k 1_{\{\tau > t_k\}} \right] + \mathbb{E} \left[ e^{-\int_{0}^{\tau} r_s \, ds} 1_{\{\tau \leq T^*\}} R(\tau) \right]$$
We assume the existence of stochastic default intensity $\lambda(t)$ which has the dynamics of a Vasicek process with time-dependent parameters under $\mathbb{Q}$:

$$d\lambda_t = \kappa_{\lambda}(\theta_{\lambda}(t) - \lambda_t)dt + \sigma_{\lambda}(t) \cdot dW^\mathbb{Q}_t,$$

The time of default is then the first arrival time of a Doubly Stochastic Poisson Process:

$$\tau = \min\left\{t \mid \int_0^t \lambda(s)ds \geq E(1)\right\}$$

The default intensity was calibrated from CDS market quotes for the airline taking out the loan of interest.
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Assume that a 2-d Brownian Motion drives the Ornstein-Uhlenbeck SDEs:

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\[ dW_t^\ell \, dW_t^\lambda = \rho \, dt \]

With the model architecture in place, all expectations in the loan valuation can be computed analytically through change of measure techniques similar to those in FX options.

\[
M = \sum_{k=1}^{N} \mathbb{E} \left[ e^{-\int_{t_k}^{\tau} r_s \, ds} \, 1_{\{\tau > t_k\}} \, C_k \right] \\
+ \sum_{k=1}^{N} \mathbb{E} \left[ 1_{\{t_{k-1} < \tau \leq t_k\}} \, e^{-\int_{0}^{\tau} r_s \, ds} \left( 1 + (\tau - t_{k-1}) L \right) P_{k-1} \right] \\
- (1 - \pi) \sum_{k=1}^{N} \mathbb{E} \left[ e^{-\int_{0}^{\tau} r_t \, dt} \, 1_{\{t_{k-1} < \tau \leq t_k\}} \left( K_k(\tau) - S(\tau) \right)^+ \right]
\]

where \( L \) is the loan rate and \( K_k(\tau) = P_{k-1} + (\tau - t_{k-1}) L \, P_{k-1} + TC \).
Enhanced Equipment Trust Certificates (EETCs)

- A form of corporate bond
  - Secured by a pool of aircraft, which are purchased by the airline with the proceeds
  - Rated (often significantly) higher than the underlying airline’s unsecured debt
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- Features
  - Constant levels of over-collateralization
  - Perfected security interest in the aircraft that is enforceable in a timely manner
  - Section 1110 protection: automatic 60 day repossession of aircraft outside of normal bankruptcy proceedings
  - Parsed into multiple tranches, each with their own LTV ratio, loan rate, amortization schedule, maturity, credit rating and seniority (senior, mezzanine, junior/subordinate certificates)
Terms of EETC issued by Continental Airlines in 2007

<table>
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  - S&P: **B**
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- Low LTV + perfected security interest leads to higher ratings for tranches:

  Aircraft appraisal: $1,146,810,000 → LTV = 73.79%
Valuing EETCs within the Framework

- The recovery payment for the senior tranche is determined the same way as for a standard loan. The recovery for the next tranche is determined similarly but now we need to subtract the recovery paid to the senior tranche from the proceeds of the sale of the aircraft. Similarly, we may determine the recovery paid out to more junior tranches.
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- With a slight modification to the general loan valuation formula presented earlier because of the priority given in recovery, the EETC valuation is almost exactly the same as a standard loan:

\[
M = \sum_{i=A,B,C} \sum_{k=1}^{N} \mathbb{E} \left[ e^{-\int_{t_{k}}^{\tau} r_s ds} 1_{\{\tau > t_k\}} C_k^i \right] 
+ \sum_{i=A,B,C} \sum_{k=1}^{N} \mathbb{E} \left[ 1_{\{t_{k-1} < \tau \leq t_k\}} e^{-\int_{0}^{\tau} r_s ds} (1 + (\tau - t_{k-1})L_i) P_{k-1}^i \right] 
- (1 - \pi) \sum_{k=1}^{N} \mathbb{E} \left[ e^{-\int_{0}^{t_{k-1}} r_t dt} 1_{\{t_{k-1} < \tau \leq t_k\}} (K_{k}^{(ABC)}(\tau) - S(\tau))^{+} \right] 
\]

where \( K_{k}^{(ABC)}(\tau) = TC + \sum_{i=A,B,C} (1 + (\tau - t_{k-1})L_i) P_{k-1}^i \)
As an example, we implemented the analytical loan valuation model as applied to the 2007 EETC issued by Continental Airlines.

- **Features**
  - Collateralized by 12 Boeing 737-824 aircraft and 18 Boeing 737-924ER, delivered new.
  - 3 Tranches: A, B, C

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  - Recovery Rate of $\pi = 0.19$ as estimated from airline industry historical default studies.

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The calibration results yielded that \( \rho = 0.72, \eta = 0.52 \) for the 2007 EETC issued by Continental Airlines. Due to the risk adjustment, the equipment price depreciates faster under the risk-neutral measure \( \mathbb{Q} \) than under the physical measure \( \mathbb{P} \). The risk adjusted residual value curve (RA-RVC) corresponding to the market price of risk parameter \( \eta = 0.52 \) is: