Components of bull and bear markets: bull corrections and bear rallies

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INTRO: some questions that motivated us to pursue this topic

TRADITIONAL: Ex post dating algorithms (filters) for bull and bear markets

NEW: probability model for the distribution of aggregate stock returns
  - focused to identify bear market rallies and bull market corrections

OUTLINE:

Proposed model structure

Bayesian estimation & model comparison

Some results concerning implied characteristics of market dynamics
  - Characteristics of market trends and sub-trends implied by parameter estimates
  - Identification of turning points

Applications: predicting turning points and VaR

Recent market conditions
Motivation: Questions

- Are there low frequency trends in stock returns?
  - Can we identify trends or cycles in aggregate stock returns?
  - Can they be used to improve investment decisions?

- Are two ’regimes’ adequate to capture the dynamics?
  - What are typical characteristics of bull and bear market regimes?
  - Is it useful to model intra-regime dynamics?

- Is it useful to invest in a probabilistic approach?
  - What is the probability that this a bull market rather than a bear rally?
  - What is the probability of moving from a bull correction into a bear?

- Is it useful to use information in the entire distribution of returns?
  - Do investors use both return & and risk to identify the state or regime?
  - How do bear rallies and bull markets differ?
Some Contributions:

We propose a new 4-state Markov-switching (MS) model for stock returns that:

- Allows bull and bear regimes to be unobserved and stochastic
- Accommodates bear rally and bull correction states within regimes
  - Bear and bear rally states govern the bear regime
  - Bull and bull correction states govern the bull regime
- Captures heterogeneous intra-regime dynamics
  - Allow for bear rallies and bull corrections without a regime change
  - Realized bull and bear regimes can be different over time
  - Conditional autoregressive heteroskedasticity in a regime
- Probability statements on regimes and future returns available
  - What is the probability of a bear market rally at time $t$?
  - What is the probability of a transition from a rally to a bull market?
- Can forecast (both market states and returns) out-of-sample
  - Out-of-sample forecasts useful for market timing
  - Conditional VaR predictions are sensitive to market regimes
Applications of MS models to stock returns include, among many others:


- **Relate business cycles and stock market regimes:** Hamilton and Lin (1996)

- **Duration dependence in stock market cycles:** Maheu and McCurdy (2000a), Lunde and Timmermann (2004)


- **Interest rates:** Garcia and Perron (1996), Ang and Bekaert (2002b)
Data

- Daily capital gain return
  - 1885-1925 capital gain returns from Schwert (1990)
  - 1926-2008 CRSP S&P 500, vwretx
  - 2009-2010 Reuters, SPTRTN on SPX

- Convert to daily continuously compounded returns
- Compute weekly return as Wed to Wed
- Compute weekly $RV_t$ as sum of intra-week daily squared returns
- Scale by 100

<table>
<thead>
<tr>
<th>Table: Weekly Return Statistics (1885-2010)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>6498</td>
</tr>
</tbody>
</table>

$^a$ Continuously compounded returns

$^b$ Jarque-Bera normality test: p-value = 0.00000
New MS-4 Model allowing Bull Corrections and Bear Rallies

**MS-4**

\[
\begin{align*}
    r_t | s_t & \sim N(\mu_{s_t}, \sigma_{s_t}^2) \\
    p_{ij} & = p(s_t = j | s_{t-1} = i), \quad i = 1, \ldots, 4, \ j = 1, \ldots, 4.
\end{align*}
\]

Terminology & Identification:

- States refer to \( s_t \) and are identified by:
  - \( \mu_1 < 0 \) (bear state),
  - \( \mu_2 > 0 \) (bear rally state),
  - \( \mu_3 < 0 \) (bull correction state),
  - \( \mu_4 > 0 \) (bull state);
  - \( \sigma_{s_t}^2 \) No restriction

- Regimes combine states as follows:
  - \( s_t = 1, 2 \) bear regime
  - \( s_t = 3, 4 \) bull regime
MS-4 Model allowing Bull Corrections and Bear Rallies, cont.

Transition matrix \( P = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix} \)

The unconditional probabilities associated with \( P \) are defined as \( \pi_i, \ i = 1, ..., 4 \)

We impose the following conditions on long-run returns in each regime\(^1\),

\[
E[r_t \mid \text{bear regime, } s_t = 1, 2] = \frac{\pi_1}{\pi_1 + \pi_2} \mu_1 + \frac{\pi_2}{\pi_1 + \pi_2} \mu_2 < 0
\]

\[
E[r_t \mid \text{bull regime, } s_t = 3, 4] = \frac{\pi_3}{\pi_3 + \pi_4} \mu_3 + \frac{\pi_4}{\pi_3 + \pi_4} \mu_4 > 0.
\]

\(^1\)Since investors cannot identify states with probability 1, modeling investors’ expected returns at each point is beyond the scope of this paper. Regimes or states may have negative expected returns for some period for a variety of reasons such as changes in risk premiums due to learning following breaks, different investment horizons, etc.
## Bayesian Estimation

### MS-K

- $r_t | s_t \sim N(\mu_{s_t}, \sigma_{s_t}^2)$
- $p_{ij} = p(s_t = j | s_{t-1} = i), \; i = 1, \ldots, K, \; j = 1, \ldots, K.$

- 3 groups of parameters $M = \{\mu_1, \ldots, \mu_K\}$, $\Sigma = \{\sigma_1^2, \ldots, \sigma_K^2\}$, and the elements of the transition matrix $P$.
- $\theta = \{M, \Sigma, P\}$ and given data $I_T = \{r_1, \ldots, r_T\}$.
- Augment the parameter space to include the states $S = \{s_1, \ldots, s_T\}$.
- Conditionally conjugate priors $\mu_i \sim N(m_i, n_i^2)$, $\sigma_i^{-2} \sim G(v_i/2, s_i/2)$ and each row of $P$ follows a Dirichlet distribution.
Bayesian Estimation

- Gibbs sampling from the full posterior $p(\theta, S|I_T)$ by sequentially sampling
  - $S|M, \Sigma, P$
    - Joint draw of $S$ following Chib (1996) (forward-backward smoother)
  - $M|\Sigma, P, S$
    - Standard linear model results
  - $\Sigma|M, P, S$
    - Standard linear model results
  - $P|M, \Sigma, S$
    - Dirichlet draw

- Drop any draws that violate identification constraints
Bayesian Estimation

- Discard an initial set of draws to remove any dependence from startup values.
- Remaining draws \( \{S(j), M(j), \Sigma(j), P(j)\}_{j=1}^{N} \) are collected.
- Simulation consistent estimates can be obtained as sample averages of the draws.
  
  \[
  \frac{1}{N} \sum_{j=1}^{N} \mu_k^{(j)} \xrightarrow{N \to \infty} E[\mu_k|I_T], \quad \frac{1}{N} \sum_{j=1}^{N} \sigma_k^{(j)} \xrightarrow{N \to \infty} E[\sigma_k|I_T]
  \]

- Byproduct of estimation is smoothed state estimates
  
  \[
p(\hat{s}_t = i|I_T) = \frac{1}{N} \sum_{j=1}^{N} 1_{s_t = i}(S(j))
  \]
  
  for \( i = 1, \ldots, K \).
- Forecasts and estimates account for parameter and regime uncertainty.
Marginal likelihood for model $M_i$ is defined as

$$p(r|M_i) = \int p(r|M_i, \theta)p(\theta|M_i)d\theta$$

$p(\theta|M_i)$ is the prior and

$$p(r|M_i, \theta) = \prod_{t=1}^{T} f(r_t|I_{t-1}, \theta)$$

is the likelihood which has $S$ integrated out according to

$$f(r_t|I_{t-1}, \theta) = \sum_{k=1}^{K} f(r_t|I_{t-1}, \theta, s_t = k)p(s_t = k|\theta, I_{t-1}).$$
Bayes Factors

Chib (1995) estimate of the marginal likelihood

\[
p(r|\mathcal{M}_i) = \frac{p(r|\mathcal{M}_i, \theta^*)p(\theta^*|\mathcal{M}_i)}{p(\theta^*|r, \mathcal{M}_i)}
\]

where \(\theta^*\) is a point of high mass in the posterior pdf.

A log-Bayes factor between model \(\mathcal{M}_i\) and \(\mathcal{M}_j\) is defined as

\[
\log(BF_{ij}) = \log(p(r|\mathcal{M}_i)) - \log(p(r|\mathcal{M}_j)).
\]

Kass and Raftery (1995) suggest interpreting the evidence for \(\mathcal{M}_i\) versus \(\mathcal{M}_j\) as

- \(0 \leq \log(BF_{ij}) < 1\) not worth more than a bare mention
- \(1 \leq \log(BF_{ij}) < 3\) positive
- \(3 \leq \log(BF_{ij}) < 5\) strong
- \(\log(BF_{ij}) \geq 5\) very strong
State Densities

![Graph showing state densities with density on the y-axis and values from -10 to 10 on the x-axis. The graph includes curves for s=1, s=2, s=3, and s=4, each with different shapes and colors. The legend indicates the colors and labels for each curve.]
## MS-4-State Model Posterior Estimates

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>95% DI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>-0.94</td>
<td>(-1.50, -0.45)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.23</td>
<td>(0.04, 0.43)</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>-0.13</td>
<td>(-0.31, -0.01)</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>0.30</td>
<td>(0.22, 0.38)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>6.01</td>
<td>(5.41, 6.77)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>2.63</td>
<td>(2.36, 3.08)</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>2.18</td>
<td>(1.94, 2.39)</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>1.30</td>
<td>(1.20, 1.37)</td>
</tr>
<tr>
<td>$\mu_1/\sigma_1$</td>
<td>-0.16</td>
<td>(-0.25, -0.07)</td>
</tr>
<tr>
<td>$\mu_2/\sigma_2$</td>
<td>0.09</td>
<td>(0.02, 0.17)</td>
</tr>
<tr>
<td>$\mu_3/\sigma_3$</td>
<td>-0.06</td>
<td>(-0.14, -0.01)</td>
</tr>
<tr>
<td>$\mu_4/\sigma_4$</td>
<td>0.23</td>
<td>(0.17, 0.31)</td>
</tr>
</tbody>
</table>

Transition matrix $P = \begin{pmatrix} 0.921 & 0.076 & 0 & 0.003 \\ 0.015 & 0.966 & 0 & 0.019 \\ 0.010 & 0 & 0.939 & 0.051 \\ 0.001 & 0 & 0.039 & 0.960 \end{pmatrix}$
### Unconditional State Probabilities

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>95% DI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_1 )</td>
<td>0.070</td>
<td>(0.035, 0.117)</td>
</tr>
<tr>
<td>( \pi_2 )</td>
<td>0.157</td>
<td>(0.073, 0.270)</td>
</tr>
<tr>
<td>( \pi_3 )</td>
<td>0.304</td>
<td>(0.216, 0.397)</td>
</tr>
<tr>
<td>( \pi_4 )</td>
<td>0.469</td>
<td>(0.346, 0.579)</td>
</tr>
</tbody>
</table>

- Unconditional prob of bear \( \pi_1 + \pi_2 = 0.227 \)
- Unconditional prob of bull \( \pi_3 + \pi_4 = 0.773 \)
### Some Posterior Regime Statistics for Bear Markets

<table>
<thead>
<tr>
<th></th>
<th>MS-2</th>
<th>MS-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance from $\text{Var}(E[r_t</td>
<td>s_t]</td>
<td>s_t=1,2)$</td>
</tr>
<tr>
<td>variance from $E[\text{Var}(r_t</td>
<td>s_t)</td>
<td>s_t=1,2]$</td>
</tr>
<tr>
<td>skewness</td>
<td>$0$</td>
<td>$-0.42$</td>
</tr>
<tr>
<td>kurtosis</td>
<td>$3$</td>
<td>$5.12$</td>
</tr>
</tbody>
</table>

Analogous results for bull markets are in the paper.
### Posterior Statistics for Regimes and States

<table>
<thead>
<tr>
<th></th>
<th>posterior mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear mean</td>
<td>-0.13</td>
</tr>
<tr>
<td>Bear duration</td>
<td>77.8</td>
</tr>
<tr>
<td>Bear cumulative return</td>
<td>-9.94</td>
</tr>
<tr>
<td>Bear stdev</td>
<td>4.04</td>
</tr>
<tr>
<td>Bull mean</td>
<td>0.13</td>
</tr>
<tr>
<td>Bull duration</td>
<td>256.0</td>
</tr>
<tr>
<td>Bull cumulative return</td>
<td>33.0</td>
</tr>
<tr>
<td>Bull stdev</td>
<td>1.71</td>
</tr>
<tr>
<td>s=1: cumulative return</td>
<td>-12.4</td>
</tr>
<tr>
<td>s=2: cumulative return</td>
<td>7.10</td>
</tr>
<tr>
<td>s=3: cumulative return</td>
<td>-2.13</td>
</tr>
<tr>
<td>s=4: cumulative return</td>
<td>7.88</td>
</tr>
<tr>
<td>s=1: duration</td>
<td>13.5</td>
</tr>
<tr>
<td>s=2: duration</td>
<td>31.2</td>
</tr>
<tr>
<td>s=3: duration</td>
<td>17.9</td>
</tr>
<tr>
<td>s=4: duration</td>
<td>27.2</td>
</tr>
</tbody>
</table>

Posterior statistics for various population moments
Log Marginal Likelihoods: Alternative Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\log f(Y \mid \text{Model})$</th>
<th>log-Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant mean with constant variance</td>
<td>-14924.1</td>
<td>1183.7</td>
</tr>
<tr>
<td>Constant mean with 4-state i.i.d variance</td>
<td>-14256.7</td>
<td>516.3</td>
</tr>
<tr>
<td>MS-2-state mean with 4-state i.i.d. variance</td>
<td>-14009.5</td>
<td>269.1</td>
</tr>
<tr>
<td>MS-2-state mean with coupled MS 2-state variance</td>
<td>-13903.3</td>
<td>162.9</td>
</tr>
<tr>
<td>MS-4-state mean with coupled MS 2-state variance</td>
<td>-13849.9</td>
<td>109.5</td>
</tr>
<tr>
<td>MS-4-state mean with coupled MS 4-state variance</td>
<td>-13740.4</td>
<td></td>
</tr>
</tbody>
</table>

- MS-4 strongly dominates all alternatives
LT dating algorithm, MS-4 and MS-2 Smoothed Probabilities

Maheu-McCurdy-Song (University of Toronto)
Components of bull and bear markets
MS-4, 1980-1985

Components of bull and bear markets

Maheu-McCurdy-Song (University of Toronto)
Components of bull and bear markets
Components of bull and bear markets
Summary

- Sorting of states and regimes is precise

- Bull and bear regime are heterogeneous
  - Bear regimes feature recurrence of states 1 (bear) and 2 (bear rally)
  - Bull regimes feature recurrence of states 3 (bull correction) and 4 (bull)

- Most turning points occur through bear rally or bull correction
  - $p(s_t = 2|s_{t+1} = 4, s_t = 1 \text{ or } 2) = 0.9342$
  - $p(s_t = 3|s_{t+1} = 1, s_t = 3 \text{ or } 4) = 0.8663$

- Asymmetric transitions both within regimes and between regimes
  - Bull corrections revert to bull more often than bear rallies bounce back to bear
Predictive Density of Returns

The predictive density for future returns based on current information at time \( t - 1 \) is computed as

\[
p(r_t|I_{t-1}) = \int f(r_t|\theta, I_{t-1}) p(\theta|I_{t-1}) d\theta
\]

which involved integrating out both state and parameter uncertainty using the posterior distribution \( p(\theta|I_{t-1}) \). From the Gibbs sampling draws \( \{S(j), M(j), \Sigma(j), P(j)\}_{j=1}^{N} \) based on data \( I_{t-1} \) we approximate the predictive density as

\[
p(\hat{r}_t|I_{t-1}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=0}^{K} f(r_t|\theta^{(i)}, I_{t-1}, s_t = k) p(s_t = k|s_{t-1}^{(i)}, \theta^{(i)})
\]

where \( f(r_t|\theta^{(i)}, I_{t-1}, s_t = k) \) follows \( N(\mu_k^{(i)},\sigma_k^{2(i)}) \) and \( p(s_t = k|s_{t-1}^{(i)}, \theta^{(i)}) \) is the transition probability.
Value-at-Risk (VaR)

- $\text{VaR}(\alpha),t$ is 100$\alpha$ percent quantile for the distribution of $r_t$ given $I_{t-1}$.
- Compute $\text{VaR}(\alpha),t$ from the predictive density MS-4 model as
  
  $$p(r_t < \text{VaR}(\alpha),t | I_{t-1}) = \alpha.$$ 

- Given a correctly specified model, the prob of a return of $\text{VaR}(\alpha),t$ or less is $\alpha$.
- Comparison with $N(0, s^2)$ where $s^2$ is the sample variance using $I_{t-1}$.
Out-of-Sample VaR and Probability of Bull
Recent State of the Aggregate Market

Data to close of Jan. 20, 2010

- $p(s_t = 1|I_t) = 0.0008$ bear
- $p(s_t = 2|I_t) = 0.0714$ bear rally
- $p(s_t = 3|I_t) = 0.0633$ bull correction
- $p(s_t = 4|I_t) = 0.8645$ bull

Following week, transition to a bull market correction
Propose a new 4-state Markov-switching (MS) model for stock returns
Offers richer characterizations of market dynamics
  - Two states govern the bear regime
  - Two states govern the bull regime

Heterogeneous intra-regime dynamics
  - Allow for bear rallies and bull corrections without a regime change
  - Realized bull and bear regimes can be different over time
  - Conditional autoregressive heteroskedasticity in a regime

Probability statements on regimes and future returns available
Our model strongly dominates other alternatives
Estimated bull and bear regimes match traditional sorting algorithms
Bull corrections and bear rallies empirically important
Out-of-sample forecasts of turning points
VaR predictions sensitive to market regimes