Optimal Portfolio Choice with Contagion Risk and Restricted Information

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Contagion Risk

Contagion-Inducing Event

Stock A

Stock B

Calm

Contagion

Calm
Introduction

How to deal with contagion risk in an asset allocation model?

- **Starting point:** asset allocation in a jump-diffusion setup
  - Merton (1969, 1971), Liu/Pan (2003), Liu/Longstaff/Pan (2003), Branger/Schlag/Schneider (2008), . . .

- **First extension:** joint Poisson jumps
  - Das/Uppal (2004), Kraft/Steﬀensen (2008), Ait-Sahalia/Cacho-Diaz/Hurd (2009), . . .
  - disregard the time dimension of contagion

- **Second extension:** regime-switching models
  - state variable and asset prices are not linked directly
  - up to now, mainly diffusion models
How to deal with contagion risk in an asset allocation model?

**Our approach**
- Two economic regimes ('calm', 'contagion')
- Regime switches and asset prices are linked directly: some (not all) asset price jumps trigger contagion
- **Explicitly takes time dimension of contagion into account**
- See Branger, Kraft, Meinerding (2009) (focus on model risk)

**Restricted information**
- Investor cannot identify the state directly (... but has to learn from historical asset prices)
- (Subjective) probability of being in the calm state:

\[ \hat{p}_t \in [0, 1] \]

- Investor optimizes conditional upon the **state variable** \( \hat{p}_t \)
Contagion Risk and Restricted Information

Branger, Kraft, Meinerding
Introduction

Main Contributions

1. Contagion and learning have a substantial impact
   - underreaction to contagion-triggering jumps
   - overreaction to noncontagious jumps
     (and subsequent re-adjustment of portfolio)

2. Complete and incomplete market differ structurally
   - complete market: largest reaction to first jump
     ('risk of contagion')
   - incomplete market: largest reaction to subsequent jumps
     ('confirmation of contagion')
   - larger trading volume in complete market

3. Significant hedging demand
   - up to 50% of speculative demand
   - may be nonmonotonic function of state variable $\hat{p}_t$
Two risky assets (A and B) with dynamics

\[
\frac{dS_i(t)}{S_i(t)} = \mu_i Z(t) \, dt + \sigma_i Z(t) \, dW_i(t) - \sum_{K \neq Z(t-)} L_i^{Z(t-),K} \, dN^K(t)
\]

under the 'large' filtration \( \{\mathcal{F}_t\}_{t \in [0,T]} \)

\( Z(t) \): current state of the economy (calm/contagion)

- **Riskless asset** (constant interest rate \( r \))
- **Derivatives** (only if needed for market completeness)
- **Economy switches between 2 states** ('calm', 'contagion')
  - two types of jumps
    - 1. jump induces loss in one asset
    - 2. jump induces loss in one asset and triggers contagion
  - overall jump intensity larger in contagion state (reflecting turbulence in the market)
  - constant loss size for each sort of jump
  - \( N^K \) counts number of jumps into state \( K \)
Investor

- can perfectly distinguish jumps and diffusion
- ... but cannot distinguish the different types of jumps
- filters a subjective probability of the calm state \( \hat{p}_t \) out of historical asset prices
- decides on his optimal portfolio using the 'small' filtration \( \{G_t\}_{t \in [0, T]} \subset \{F_t\}_{t \in [0, T]} \)
- CRRA utility (with RRA \( \gamma = 3 \) in the benchmark case)
- maximizes utility from terminal wealth only
- investment horizon: 5 years (in the benchmark case)

Complete Market

- investor chooses exposures against the four risk factors (which he can distinguish with restricted information)
- investor uses derivatives to disentangle the risk factors

Incomplete Market

- investor chooses portfolio weights for the two risky assets
- investor has to accept the whole package of risk factors
Numerical Results

Parametrization

- Main parameters taken from the literature (EJP 2003, BCJ 2007):
  \[ r = 0.01, \sigma = 0.15, \rho = 0.5, L = 0.04 \]

- Only jump parameters differ across both states

- Jump intensities are calibrated via
  - \( \xi \): jump intensity multiplicator calm-contagion
  - \( \alpha \): (conditional) probability of contagion-triggering jumps

- **Benchmark case** (identical assets)
  - \( \xi_i = 5, \alpha_i = 0.2 \)
  - average (unconditional) jump intensity per year: 0.62

- **Second case** (different assets)
  - \( \xi_A = 5, \alpha_A = 0.2 \) (A is more severely hit by contagion)
  - \( \xi_B = 2.5, \alpha_B = 0.5 \) (B is more likely to trigger contagion)

- **Risk Premia**
  - diffusion risk: 0.0525
  - jump risk: 0.08 (calm state) and 0.016 (contagion state)

\[ \rightarrow \text{Optimal and suboptimal filter equal} \]
1. Impact of **restricted information**
   - Noncontagious jump: **overreaction** (and subsequent correction)
   - Contagious jump: **underreaction**

2. **Complete versus incomplete market**
   - Complete market: largest reaction to first jump ('risk of contagion')
   - Incomplete market: largest reaction to subsequent jumps ('confirmation of contagion')
**Hedging Demand** for jump risk

- Worse investment opportunities in contagion state
  → positive hedging demand

- Largest probability update for $\hat{p}_t \approx 0.8$

- Largest influence of $\hat{p}_t$ on utility for $\hat{p}_t = 1$
  → largest hedging demand for $\hat{p}_t \approx 0.9$
Numerical Results

Solution of the Portfolio Planning Problem with Different Assets

Complete Market

- **Asset A**
  - heavily affected by contagion 
    \((\xi_A = 5, \alpha_A = 0.2)\)
  - largest trading volume

- **Asset B**
  - more likely to trigger contagion 
    \((\xi_B = 2.5, \alpha_B = 0.5)\)
  - induces largest portfolio adjustments

Incomplete Market

- Jump risk ‘spills over’ from asset B to asset A
Increasing **Diffusion Risk**
- no impact on complete market
- less impact of contagion in incomplete market
- differences between complete and incomplete market increase

**Loss size**
- no qualitative changes

**Investment horizon**
- utility functions flatten out with larger horizons

**Relative risk aversion**
- no qualitative changes

**Jump risk premia**
- no qualitative changes

**Average duration of the contagion regime**
- has only marginal effects
- main driver of our results:
  - Contagion is a state (not a one-time event)
Concluding Remarks

Conclusion

1. **Learning has a substantial impact**
   - **underreaction** to contagion-triggering jumps
   - **overreaction** to noncontagious jumps
   - stocks that are most hit by contagion
     \[\rightarrow\] largest trading volume
   - stocks that most likely trigger contagion
     \[\rightarrow\] induce largest portfolio adjustments

2. **Complete and incomplete market differ structurally**
   - complete market: largest reaction to 'risk of contagion'
   - incomplete market: largest reaction to 'confirmation'

3. **Significant hedging demand**
   - up to 50% of speculative demand
   - may be nonmonotonic function of state variable \(\hat{p}_t\)

**Future research**
- Analyze the difference between optimal and suboptimal filter
- General equilibrium (\[\rightarrow\] market price of contagion risk)
The Markov Chain
The Suboptimal Filter

\[
d\hat{p}_t = \left( (1 - \hat{p}_t) \lambda^{\text{cont,calm}} - \hat{p}_t (\lambda^{\text{calm,cont}}_A + \lambda^{\text{calm,cont}}_B) \right) dt \\
+ \hat{p}_t \left( \frac{\lambda^{\text{calm,calm}}_A}{\hat{\lambda}_A(\hat{p}_t)} - 1 \right) \left( d\hat{N}_A(t) - \hat{\lambda}_A(\hat{p}_t) dt \right) \\
+ \hat{p}_t \left( \frac{\lambda^{\text{calm,calm}}_B}{\hat{\lambda}_B(\hat{p}_t)} - 1 \right) \left( d\hat{N}_B(t) - \hat{\lambda}_B(\hat{p}_t) dt \right)
\]

where the estimated subjective intensity of \( \hat{N}_i \) equals

\[
\hat{\lambda}_i(\hat{p}_t) = \hat{p}_t \left( \lambda^{\text{calm,calm}}_i + \lambda^{\text{calm,cont}}_i \right) + (1 - \hat{p}_t) \lambda^{\text{cont,cont}}_i
\]
\begin{align*}
dp_t &= \rho_t(1 - \rho_t) \left[ \lambda_A^{\text{cont}, \text{cont}} + \lambda_B^{\text{cont}, \text{cont}} - \lambda_A^{\text{calm}, \text{calm}} - \lambda_B^{\text{calm}, \text{calm}} - \lambda_A^{\text{calm}, \text{cont}} - \lambda_B^{\text{calm}, \text{cont}} \right] dt \\
&+ (1 - \rho_t) \lambda^{\text{cont}, \text{calm}} dt \\
&+ \rho_t(1 - \rho_t) \left[ \frac{\left( \mu_A^{\text{calm}} \right)^2 - \left( \mu_A \right)^2}{(1 - \rho^2)\sigma_A^2} + \frac{\left( \mu_B^{\text{calm}} \right)^2 - \left( \mu_B \right)^2}{(1 - \rho^2)\sigma_B^2} - 2\rho \frac{\mu_A^{\text{calm}}\mu_B^{\text{calm}} - \mu_A\mu_B^{\text{cont}}}{(1 - \rho^2)\sigma_A\sigma_B} \\
&+ \frac{(1 - \rho_t)(\mu_A^{\text{cont}})^2 - \rho_t(\mu_B^{\text{calm}})^2}{\sigma_A^2} + \frac{(1 - \rho_t)(\mu_B^{\text{cont}})^2 - \rho_t(\mu_B^{\text{calm}})^2}{(1 - \rho^2)\sigma_B^2} \left(1 - \rho \frac{\sigma_B}{\sigma_A} \right)^2 dt \\
&+ \rho_t(1 - \rho_t) \left[ \frac{\mu_A^{\text{calm}} - \mu_A}{\sigma_A} dW_t^A + \frac{\mu_B^{\text{calm}} - \mu_B}{\sigma_B} dW_t^B \right] \\
&+ \left( \frac{\lambda_A^{\text{calm}, \text{calm}} \rho_t - \lambda_A^{\text{cont}, \text{cont}} (1 - \rho_t) - (\lambda_A^{\text{calm}, \text{calm}} + \lambda_A^{\text{calm}, \text{cont}})\rho_t}{\lambda_A^{\text{calm}, \text{calm}}} \right) dN_{t}^{A, \text{obs}} \\
&+ \left( \frac{\lambda_B^{\text{calm}, \text{calm}} \rho_t - \lambda_B^{\text{cont}, \text{cont}} (1 - \rho_t) - (\lambda_B^{\text{calm}, \text{calm}} + \lambda_B^{\text{calm}, \text{cont}})\rho_t}{\lambda_B^{\text{calm}, \text{calm}}} \right) dN_{t}^{B, \text{obs}}
\end{align*}
Backup

Optimization problem in a complete or incomplete market

\[ G(t, X_t, \hat{p}_t) = \max_{\Pi \in \mathcal{A}(t, \hat{p}_t)} \{ E[u(X_T)|\hat{p}_t] \} \]

s.t. \[ \frac{dX_t}{X_t} = rdt \]
\[ + \theta^\text{diff}_A(t, \hat{p}_t) \cdot (d\hat{W}_A(t) + \hat{\eta}^\text{diff}_A dt) \]
\[ + \theta^\text{diff}_B(t, \hat{p}_t) \cdot (d\hat{W}_B(t) + \hat{\eta}^\text{diff}_B dt) \]
\[ + \theta^\text{jump}_A(t, \hat{p}_t) \left[ d\hat{N}_A(t) - \lambda_A(\hat{p}_t)dt - \hat{\eta}^\text{jump}_A(\hat{p}_t)\lambda_A(\hat{p}_t)dt \right] \]
\[ + \theta^\text{jump}_B(t, \hat{p}_t) \left[ d\hat{N}_B(t) - \lambda_B(\hat{p}_t)dt - \hat{\eta}^\text{jump}_B(\hat{p}_t)\lambda_B(\hat{p}_t)dt \right] \]

or \[ \frac{dX(t)}{X(t)} = \pi_A(t, \hat{p}_t) \frac{dS_A(t)}{S_A(t)} + \pi_B(t, \hat{p}_t) \frac{dS_B(t)}{S_B(t)} \]
\[ + [1 - \pi_A(t, \hat{p}_t) - \pi_B(t, \hat{p}_t)] rdt \]
Complete Market System of PDAEs

\[
\begin{align*}
    f_t(t, \hat{p}_t) + f(t, \hat{p}_t) \cdot (D + E) + f_p(t, \hat{p}_t) \cdot B \\
    + \left(1 + \theta_A^{jump}\right)^{1-\gamma} \lambda_A f(t, \hat{p}_A^+) + \left(1 + \theta_B^{jump}\right)^{1-\gamma} \lambda_B f(t, \hat{p}_B^+) &= 0 \\
    -f(t, \hat{p}_t) \cdot (1 + \hat{\eta}_A^{jump}) + f(t, \hat{p}_A^+) \cdot \left(1 + \theta_A^{jump}\right)^{-\gamma} &= 0 \\
    -f(t, \hat{p}_t) \cdot (1 + \hat{\eta}_B^{jump}) + f(t, \hat{p}_B^+) \cdot \left(1 + \theta_B^{jump}\right)^{-\gamma} &= 0
\end{align*}
\]

- \(B, D\) and \(E\) depend on the model parameters, \(\hat{p}_t\) and \(\theta_i^{jump}\)
- \(\hat{p}_i^+ = \frac{\lambda_i^{calm, calm}}{\hat{\lambda}_i} \cdot \hat{p}_t\) denotes the updated probability after a jump in stock \(i\)
Incomplete Market System of PDAEs

\[ f_t(t, \hat{p}_t) + f(t, \hat{p}_t) \cdot \left[ (1 - \gamma) \cdot A^* - 0.5\gamma(1 - \gamma) \cdot C^* - \hat{\lambda}_A - \hat{\lambda}_B \right] \\
+ f_p(t, \hat{p}_t) \cdot B + \left[ (1 - \pi_A L_A)^{1-\gamma} \cdot f(t, \hat{p}_A^+) \right] \hat{\lambda}_A \\
+ \left[ (1 - \pi_B L_B)^{1-\gamma} \cdot f(t, \hat{p}_B^+) \right] \hat{\lambda}_B = 0 \\
\]

\[ f(t, \hat{p}_t) \cdot (\hat{\mu}_A - r) - \gamma\pi_B \rho \hat{\sigma}_A \hat{\sigma}_B \cdot f(t, \hat{p}_t) - \gamma\hat{\sigma}_A^2 \pi_A \cdot f(t, \hat{p}_t) \\
- L_A \cdot (1 - \pi_A L_A)^{-\gamma} \cdot f(t, \hat{p}_A^+) \cdot \hat{\lambda}_A = 0 \\
\]

\[ f(t, \hat{p}_t) \cdot (\hat{\mu}_B - r) - \gamma\pi_A \rho \hat{\sigma}_A \hat{\sigma}_B \cdot f(t, \hat{p}_t) - \gamma\hat{\sigma}_B^2 \pi_B \cdot f(t, \hat{p}_t) \\
- L_B \cdot (1 - \pi_B L_B)^{-\gamma} \cdot f(t, \hat{p}_B^+) \cdot \hat{\lambda}_B = 0 \\
\]

- \( A^*, B \) and \( C^* \) depend on the model parameters, \( \hat{p}_t \) and \( \pi_i \)
- \( \hat{p}_i^+ = \frac{\lambda_i^{\text{calm, calm}}}{\hat{\lambda}_i} \cdot \hat{p}_t \) denotes the updated probability after a jump in stock \( i \)
## Benchmark Parametrization

<table>
<thead>
<tr>
<th>Data-generating process</th>
<th>Benchmark (equal stocks)</th>
<th>Different stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_i^{\text{calm}}$, $\sigma_i^{\text{cont}}$</td>
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<tr>
<td>$L_i^{\text{calm},\text{cont}}$</td>
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</tr>
<tr>
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<table>
<thead>
<tr>
<th>Market prices of risk</th>
<th>Benchmark (equal stocks)</th>
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<tbody>
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<table>
<thead>
<tr>
<th>Risk premia</th>
<th>Benchmark (equal stocks)</th>
<th>Different stocks</th>
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<tr>
<td>diffusion risk</td>
<td>0.0525</td>
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<tr>
<td>jump risk</td>
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<td>0.08</td>
</tr>
<tr>
<td>jump risk</td>
<td>0.016</td>
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</tbody>
</table>

Contagion Risk and Restricted Information
Branger, Kraft, Meinerding
Risk Premia

- Investor knows the model and all parameters except the state of the economy
- Suboptimal filter: **from jump processes only**
  - Optimal if drift and diffusion terms equal across states
  - Resulting restrictions in the complete market

\[
\hat{\eta}_i^{\text{diff}} = \eta_i^{\text{diff, calm}} = \eta_i^{\text{diff, cont}} =: \eta_i^{\text{diff}}
\]

\[
\hat{\lambda}_i \left(1 + \hat{\eta}_i^{\text{jump}}\right) = \lambda_i^{\text{calm, calm}} \left(1 + \eta_i^{\text{calm, calm}}\right) + \lambda_i^{\text{calm, cont}} \left(1 + \eta_i^{\text{calm, cont}}\right)
= \lambda_i^{\text{cont, cont}} \left(1 + \eta_i^{\text{cont, cont}}\right)
\]

- Similar restrictions hold in the incomplete market

**Resulting jump risk premia**
- 0.08 in the calm state
- 0.016 in the contagion state

- Constant diffusion risk premium: 0.0525